T M S C A M I D D L E S C H O O L

M A T H E M A T I C S

R E G I O N A L T E S T ©

M A R C H 4 , 2 0 2 3

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 50 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading on Scantrons and Chatsworth cards.
3. If you are using a Chatsworth or Scantron card, please follow the specific instructions given at your particular meet.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators **MAY NOT** be used on this test.
8. All problems answered correctly are worth **FIVE** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2022 – 2023 TMSCA Middle School Mathematics Regional Test

1. $56\frac{1}{2} - 47\frac{4}{5}$ = _____

A. $9\frac{3}{10}$ B. $9\frac{1}{5}$ C. $8\frac{2}{5}$ D. $8\frac{4}{7}$ E. $8\frac{7}{10}$

2. 11.003 – 8.9 = _____ (nearest tenth)

A. 3.1 B. 2.9 C. 2.7 D. 2.3 E. 2.1

3. $9 \times 11 \times 13$ = _____

A. 1,287 B. 1,391 C. 1,297 D. 1,397 E. 1,227

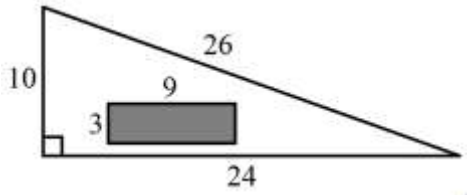
4. $2{,}456 \div (17 - 13)$ = _____

A. 588 B. 614 C. 634 D. 688 E. 606

5. A recipe calls for 6½ cups of water, 5⅓ cups of flour and ¾ cup of salt. How many cups of ingredients does the recipe need?

A. $12\frac{3}{8}$ cups B. $11\frac{5}{8}$ cups C. $11\frac{5}{12}$ cups D. $12\frac{7}{12}$ cups E. $12\frac{5}{8}$ cups

6. What is the area of the non-shaded region in the picture below?



A. 120 units^2 B. 93 units^2 C. 103 units^2 D. 54 units^2 E. 147 units^2

7. 55,000 milliliters = _____ hectoliters

A. 55 B. 5.5 C. 550 D. 0.055 E. 0.55

8. In a neighborhood, $\frac{7}{20}$ of the homes have exactly three trees, and the rest of the homes only have 1 tree. Which percentage is equivalent to the portion of homes with exactly one tree?

A. 65% B. 55% C. 45% D. 35% E. 75%

9. $m\angle A = 56^\circ$. What is the sum of the complement and supplement of $\angle A$?

A. 154° B. 124° C. 158° D. 134° E. 138°

10. 17 quarters + 9 dimes + 6 nickels + 10 pennies = 12 quarters + 13 dimes + 20 nickels + _____ pennies

A. 20 B. 15 C. 30 D. 25 E. 35

11. What is the unit rate of spending $153.00 for three-dozen gourmet candy bars?

A. $3.75 B. $4.75 C. $4.25 D. $3.50 E. $4.50

12. What is the sum of the distinct prime factors of the number 840?

A. 17 B. 21 C. 18 D. 23 E. 19

13. What is the remainder when the sum of 462 and 777 is divided by 9?

A. 2 B. 5 C. 3 D. 6 E. 4

14. If $a = 8$ and the side length of an equilateral triangle is $5a + 7$, what is the triangle's perimeter?
A. 65 units B. 47 units C. 147 units D. 123 units E. 141 units

15. If $3n - 7 = 20$, what is the value of $7n - 3$?
A. 40 B. 60 C. 78 D. 105 E. 96

16. How many total diagonals can be drawn inside a regular 20-sided polygon?
A. 170 B. 85 C. 140 D. 128 E. 102

17. On a map, 1.5 inches is equivalent to 240 miles. How many miles is equivalent to 9.5 inches?
A. 1,200 miles B. 1,840 miles C. 1,520 miles D. 1,260 miles E. 1,360 miles

18. Simplify: $\frac{4^2+(-8)-(-12)}{4-2^3}$
A. 2 B. 3 C. -4 D. -5 E. -6

19. Let A equal the set of numbers {18, 37, 23, 18} and B equal the set of numbers {33, 22, 14, 51}. Find the value of the positive difference of the medians of A and B.
A. 7 B. 22 C. 5 D. 4 E. 13

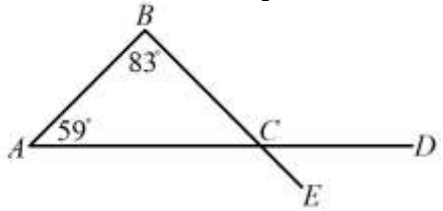
20. How many different three-digit numbers can be made using the first four prime numbers if digits can repeat?
A. 64 B. 24 C. 120 D. 12 E. 144

21. $2023^2 - 2022^2$ = _____
A. 14,161 B. 4,225 C. 4,485 D. 4,045 E. 3,127

22. What is the probability of rolling a pair of dice and getting a sum that is a prime number or greater than 9?
A. $\frac{1}{2}$ B. $\frac{17}{36}$ C. $\frac{7}{12}$ D. $\frac{23}{36}$ E. $\frac{19}{36}$

23. $1 + 2 + 3 + \ldots + 18 + 19 + 20$ = _____
A. 220 B. 210 C. 200 D. 230 E. 240

24. What is the measure of the complement to $\angle ECD$ in the picture below?



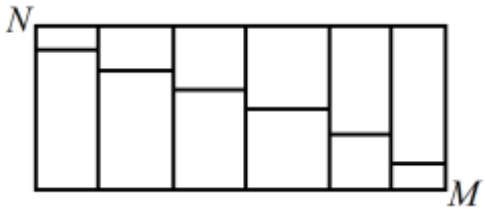
A. 46° B. 7° C. 52° D. 31° E. 38°

25. 25,861 = _____ (Roman numeral)
A. $\overline{XXV}CCCLXI$ B. $\overline{XXD}DCCCLXI$ C. $\overline{CCV}DCCCLXI$ D. $\overline{DXX}DCCCLXI$ E. $\overline{XXV}DCCCLXI$

26. 444_5 = _____ (base 7)
A. 235 B. 243 C. 227 D. 216 E. 223

27. $\{10, 20, 30, 40, 50\} \cap \{5, 10, 15, 20, 25, 30\} \cap \{6, 12, 18, 24, 30\} =$ set *A*. How many elements are in set *A*?
A. 16 B. 12 C. 1 D. 3 E. 0

28. Moving only left and/or up, how many paths exist from point *M* to point *N*?



A. 26 B. 28 C. 24 D. 20 E. 22

29. What is the geometric mean of the numbers 4, 8, and 54?
A. 12 B. $10\frac{2}{3}$ C. $10\frac{1}{3}$ D. 22 E. $24\sqrt{3}$

30. A motorcycle was clocked at a speed of 150 *mi/hr*. What was the motorcycle's speed in feet per second?
A. 200 B. 210 C. 180 D. 190 E. 220

31. The graphs of which linear equations below have a slope of ¾?
I. $8y = 6x$ II. $12x + 16y = 3$ III. $2y - 9 = 1.5x$ IV. $y = \frac{3}{4}x + 8$
A. all of them B. 1, II and III C. I, II, and IV D. II, III and IV E. I, III, and IV

32. What is the length of the line segment with endpoints of $(-10, 5)$ and $(6, -3)$?
A. $2\sqrt{65}$ units B. $4\sqrt{5}$ units C. $10\sqrt{2}$ units D. $8\sqrt{5}$ units E. $6\sqrt{2}$ units

33. The sum of three consecutive integers is 261. What is the value of five less than the least integer?
A. 82 B. 83 C. 79 D. 81 E. 93

34. What is the simple interest of depositing $2,500 into an account paying 3% after 36 months?
A. $250 B. $225 C. $200 D. $275 E. $300

35. $-|11 - 32| - |19 + (-11)| =$ _____
A. 13 B. 51 C. -29 D. -73 E. -51

36. $\frac{18.8 \times 10^{12}}{8 \times 10^{-3}} =$ _____ (scientific notation)
A. 2.35×10^{9} B. 2.35×10^{-9} C. 235×10^{15} D. 2.35×10^{15} E. 2.35×10^{-36}

37. $225^{\circ} =$ _____ (radians)
A. $\frac{5\pi}{4}$ B. $\frac{3\pi}{2}$ C. $\frac{7\pi}{5}$ D. $\frac{5\pi}{2}$ E. $\frac{7\pi}{6}$

38. If $(2x - 11)(7x + 3) = 14x^2 + Bx - 33$, then find the value of $-2B$.
A. 154 B. 66 C. 166 D. 168 E. 142

39. What is the rate of decay of the exponential decay function $f(x) = 3\left(\frac{3}{5}\right)^{x}$?
A. 60% B. 20% C. 30% D. 50% E. 40%

40. If x varies directly as y, and $x = 14$ when $y = 68$, find the value of x when $y = 408$.
A. 92 B. 88 C. 84 D. 82 E. 78

41. Steve and Joey are brothers. Steve can mow their lawn by himself in 50 minutes and Joey can mow their lawn by himself in 30 minutes. If they work together, how long would it take the brothers to mow their lawn?
A. 17.25 minutes B. 18.75 minutes C. 16.5 minutes D. 18.25 minutes E. 16.25 minutes

42. Let S equal the sum of the roots of $0 = 4x^2 + 28x$, and let P equal the product of the roots of $0 = 12 - 2x^2$. Find the value of $P - S$.
A. 1 B. -4 C. -2 D. 6 E. 3

43. What is the inverse function of the function $f(x) = \frac{1}{x+2}$?
A. $f^{-1}(x) = x - 2$ B. $f^{-1}(x) = x + 2$ C. $f^{-1}(x) = \frac{1}{x} - 2$ D. $f^{-1}(x) = \frac{1}{x} + 2$ E. $f^{-1}(x) = \frac{x-1}{2}$

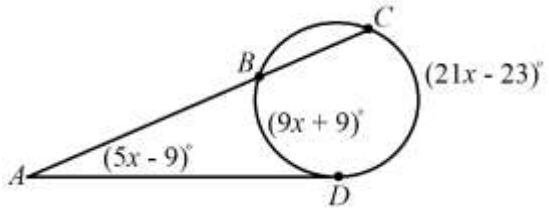
44. What is the area of a scalene triangle with side lengths measuring 4 inches, 13 inches, and 15 inches?
A. 36 in^2 B. 24 in^2 C. 32 in^2 D. 28 in^2 E. 18 in^2

45. If $\frac{2}{3}x + \frac{5}{2} = \frac{3}{2} + \frac{1}{3}x$, what is the value of $-7x$?
A. -49 B. 35 C. -28 D. 21 E. 14

46. If the solution to the system of linear equations $\begin{cases} 3a - b + 12c = 9 \\ 2a + b - 8c = -10 \\ 4a - b - 4c = -8 \end{cases}$ is (a, b, c), then what is the sum of $a + b + c$?
A. -3 B. -7 C. 1 D. -1 E. 0

47. $\left(\frac{18a^4b^{-3}}{2a^{-2}b^{-5}}\right) \cdot \left(\frac{a^{-1}b^5}{3a^3b}\right) \div \left(\frac{6a^7b^5}{2ab^{10}}\right) =$ _____
A. $\frac{b^{11}}{a^4}$ B. $\frac{3b^{11}}{a^4}$ C. $9a^8b$ D. $3a^8b$ E. $\frac{9a^8}{b}$

48. In the picture below, $m\angle A = (5x - 9)^\circ$, measure of arc $BD = (9x + 9)^\circ$, and measure of arc $CD = (21x - 23)^\circ$. What is the measure of arc BD?



A. 81° B. 72° C. 99° D. 108° E. 90°

49. Simplify: $\frac{8}{2+\sqrt{3}}$
A. $\frac{8-\sqrt{3}}{2}$ B. $\frac{16-\sqrt{3}}{2}$ C. $16 - 8\sqrt{3}$ D. $16 + 8\sqrt{3}$ E. $8 + 4\sqrt{3}$

50. What is the volume of the sphere with the equation $(x - 9)^2 + (y - 12)^2 + (z + 6)^2 - 475 = -219$, with $\pi = 3$?
A. 16,384 units^3 B. 3,072 units^3 C. 6,144 units^3 D. 12,288 units^3 E. 24,576 units^3

2022 – 2023 TMSCA Middle School Mathematics Regional Test Answer Key

1. E	18. D	35. C
2. E	19. A	36. D
3. A	20. A	37. A
4. B	21. D	38. E
5. D	22. E	39. E
6. B	23. B	40. C
7. E	24. C	41. B
8. A	25. E	42. A
9. C	26. A	43. C
10. D	27. C	44. B
11. C	28. B	45. D
12. A	29. A	46. E
13. D	30. E	47. A
14. E	31. E	48. B
15. B	32. D	49. C
16. A	33. D	50. A
17. C	34. B	

2022 – 2023 TMSCA Middle School Mathematics Regional Test Selected Answers

6. To find the area of the non-shaded region, subtract the area of the rectangle from the area of the triangle. The area of the triangle is $A = \frac{bh}{2} = \frac{10(24)}{2} = \frac{240}{2} = 120$ units^2. The area of the shaded rectangle is $A = bh = 9(3) = 27$ units^2. Therefore, the area of the non-shaded region is equal to 120 – 27 = 93 units^2.

8. If $\frac{7}{20}$ of the homes have exactly three trees, then the homes with only 1 tree equals $\frac{13}{20}$ of the homes. Since $\frac{13}{20} = \frac{65}{100}$, 65% of the homes have only one tree.

18. Using order of operations, $\frac{4^2+(-8)-(-12)}{4-2^3} = \frac{16+(-8)-(-12)}{4-8} = \frac{16-8+12}{-4} = \frac{20}{-4} = -5.$

23. The formula for finding the sum of consecutive integers is $\frac{N(F+L)}{2}$, where N = number of integers, F = 1st integer, and L = last integer. In 1 + 2 + 3 + … + 18 + 19 + 20, N = 20, F = 1, and L = 20. Therefore, the sum of 1 + 2 + 3 + … + 18 + 19 + 20 is equal to $\frac{20(1+20)}{2} = \frac{20(21)}{2} = \frac{420}{2} = 210.$

27. $\{10, 20, 30, 40, 50\} \cap \{5, 10, 15, 20, 25, 30\} = \{10, 20, 30\}$. $\{10, 20, 30\} \cap \{6, 12, 18, 24, 30\} = \{30\}$. Therefore, $A = \{30\}$, which has 1 element.

29. The geometric mean of three numbers, a, b, and c is found by the formula $\sqrt[3]{abc}$. Therefore, the geometric mean of the numbers 4, 8, and 54 is equal to $\sqrt[3]{4(8)(54)} = \sqrt[3]{1728} = 12.$

35. Using order of operations, $-|11 - 32| - |19 + (-11)| = -|-21| - |8| = -21 - 8 = -29.$

39. An exponential decay function is in the form $y = a(1 - r)^x$, where a is the initial amount and r is the rate. In the function $f(x) = 3\left(\frac{3}{5}\right)^x$, the rate of decay is therefore, $1 - r = \frac{3}{5}$, so $r = 1 - \frac{3}{5} = \frac{2}{5}$ = 40%.

43. $f(x) = \frac{1}{x+2}$ can be rewritten as $y = \frac{1}{x+2}$. To find the inverse of a function, switch x and y and solve for y. Switching x and y gives the equation $x = \frac{1}{y+2}$. To solve for y, first multiply both sides of the equation by $y + 2$ to get $x(y + 2) = 1$. Next, divide both sides of the equation by x to get $y + 2 = \frac{1}{x}$. Finally, subtract 2 from both sides of the equation to get $y = \frac{1}{x} - 2$. Therefore, the inverse function of $f(x) = \frac{1}{x+2}$ is $f^{-1}(x) = \frac{1}{x} - 2$.

41. With "work" problems, if one person does a job in x minutes and the other does the same job in y minutes, then working together they can finish the job in $\frac{xy}{x+y}$ minutes. So, if Steve can mow the lawn 50 minutes and Joey can mow the in 30 minutes, then working together, it will take them $\frac{50(30)}{50+30} = \frac{1500}{80} = \frac{75}{4} = 18.75$ minutes.

49. The conjugate of $2 + \sqrt{3}$ is $2 - \sqrt{3}$, because $\left(2 + \sqrt{3}\right)\left(2 - \sqrt{3}\right) = 1$. To simplify $\frac{8}{2+\sqrt{3}}$, you must rationalize the denominator by multiplying $\frac{8}{2+\sqrt{3}}$ by $\frac{2-\sqrt{3}}{2-\sqrt{3}}$ to get $\frac{8}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{16-8\sqrt{3}}{1} = 16 - 8\sqrt{3}.$