

**TMSCA MIDDLE SCHOOL
MATHEMATICS
STATE TEST ©
APRIL 1, 2023**

GENERAL DIRECTIONS

1. About this test:
 - A. You will be given 40 minutes to take this test.
 - B. There are 50 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading on Scantrons and Chatsworth cards.
3. If you are using a Chatsworth or Scantron card, please follow the specific instructions given at your particular meet.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators **MAY NOT** be used on this test.
8. All problems answered correctly are worth **FIVE** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2022 – 2023 TMSCA Middle School State Mathematics Championship Test

1. $768 + 382 + 6 =$ _____
 A. 1,274 B. 1,224 C. 1,156 D. 1,124 E. 1,316

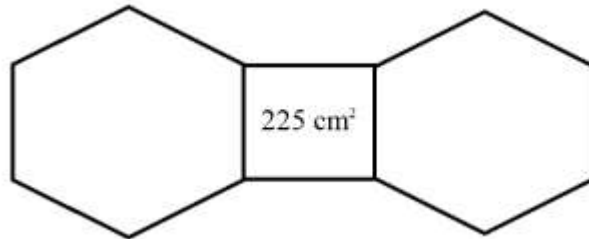
2. $1 - (0.33 - 0.07) =$ _____
 A. 1.2 B. 0.6 C. 0.66 D. 0.74 E. 0.5

3. $32\frac{1}{2} \times 2\frac{1}{5} =$ _____ (decimal)
 A. 64.1 B. 64.5 C. 71.5 D. 70.1 E. 71.1

4. $1,284 \div 1.5 =$ _____ (nearest ten)
 A. 870 B. 860 C. 850 D. 890 E. 880

5. 28,000,000 millimeters = _____ decameters
 A. 280 B. 2,800 C. 28 D. 2.8 E. 28,000

6. A square shares two of its sides with two regular hexagons to form a dodecagon, as shown below. If the area of the square is 225 cm^2 , what is the perimeter of the dodecagon?



A. 2,700 cm B. 180 cm C. 210 cm D. 195 cm E. 150 cm

7. 20% of 800 = 80% of _____
 A. 160 B. 220 C. 240 D. 180 E. 200

8. $4.0\overline{6} =$ _____ (mixed number)
 A. $4\frac{1}{15}$ B. $4\frac{2}{33}$ C. $4\frac{2}{3}$ D. $4\frac{1}{18}$ E. $4\frac{1}{12}$

9. 8 is the GCF of which pair of numbers?
 A. 16 and 64 B. 4 and 20 C. 24 and 72 D. 16 and 56 E. 64 and 96

10. What is the total number of diagonals that can be drawn inside a regular 16-sided polygon?
 A. 152 B. 126 C. 188 D. 112 E. 104

11. For her lunch, Sienna can choose from 5 different sandwiches, 3 kinds of salads, a roll or a bag of chips, and 4 different drinks. If Sienna must choose one sandwich, one salad, one bag of chips or a roll, and one drink, how many lunch combinations are possible?
 A. 120 B. 60 C. 14 D. 40 E. 80

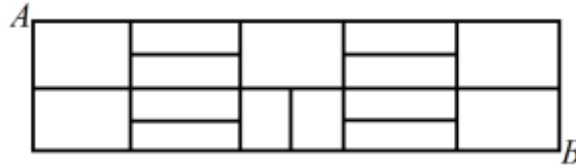
12. Ian ran ten laps on Monday, twelve laps on Tuesday, six on Wednesday, eighteen on Thursday, and fourteen laps on Friday. What was the average number of laps Ian ran for the five days?
 A. 8 B. 11 C. 10 D. 12 E. 9

13. $m\angle A$ is twelve degrees less than the supplement of $\angle B$. If $m\angle B = 134^\circ$, what is the measure of the complement of $\angle A$?
 A. 46° B. 56° C. 68° D. 44° E. 58°

14. Which lists of angle measures could be the angles measures of a triangle?

- I. $70^\circ, 25^\circ, 125^\circ$ II. $50^\circ, 55^\circ, 75^\circ$ III. $80^\circ, 95^\circ, 15^\circ$ IV. $20^\circ, 25^\circ, 135^\circ$
 A. I and III B. I, II, and III C. II and III D. II and IV E. III and IV

15. Moving only to the right and/or down, how many paths exist from point A to point B ?



- A. 40 B. 30 C. 38 D. 42 E. 36

16. Simplify: $|17 - 30| - 3^2 + (-11)$

- A. -33 B. 11 C. -11 D. 3 E. -7

17. What is the unit rate if 8 dinosaurs cost \$95.36?

- A. \$11.92 B. \$11.84 C. \$12.14 D. \$12.02 E. \$11.96

18. Delfina is thinking of two positive integers in the ratio of 11:14 that sum to 275. What is the value of the larger integer Delfina is thinking of?

- A. 121 B. 148 C. 154 D. 176 E. 198

19. What is the surface area of a cube with an inner diagonal of 16 mm?

- A. 32 mm^2 B. 64 mm^2 C. 128 mm^2 D. 256 mm^2 E. 512 mm^2

20. Point A is located at $(-4, -7)$. If point A is translated to the right nine units, up ten units, and then reflected across the x -axis, what is the product of the new coordinates of point A ?

- A. -12 B. -15 C. -8 D. 28 E. 12

21. Find the value of $a + b + c$, if $6,500 = 2^a \times 5^b \times 13^c$.

- A. 9 B. 5 C. 8 D. 6 E. 7

22. What is the next term of the sequence 0.15, 0.45, 1.35, 4.05, ...?

- A. 7.45 B. 11.25 C. 12.15 D. 12.05 E. 9.65

23. Ari asked sixty-one people if they like corn or potatoes. Ari's results showed that forty-four people liked corn, twenty-eight liked potatoes, and eleven liked both corn and potatoes. How many people that Ari questioned liked corn, but not potatoes?

- A. 31 B. 33 C. 17 D. 27 E. 29

24. If $A = 1, B = 2, \dots, Y = 25$, and $Z = 26$, what is the sum of the letters of the phrase *STATE MATH TEST*?

- A. 167 B. 171 C. 173 D. 169 E. 166

25. The base of a prism has 20 sides. What is the sum of the number of faces, vertices and edges of the prism?

- A. 122 B. 136 C. 126 D. 130 E. 138

26. $342 \times 202 =$ _____ (Roman numeral)

- A. $\overline{\text{LXIXLXXXIV}}$ B. $\overline{\text{LXIXCCCXL}}$ C. $\overline{\text{LXIXCCCXLIV}}$ D. $\overline{\text{LXIXCXXXIV}}$ E. $\overline{\text{LXIXLXXXIX}}$

27. What is the degree of the polynomial $5m^4 + 3m^2 + 7mn + 9n - 18$?

- A. 9 B. 10 C. 8 D. 6 E. 4

28. If 1 horse + 5 cows = 1 chicken and 3 horses + 2 cows = 2 chickens, how many cows are equal to 1 chicken?

- A. 6 B. 9 C. 13 D. 7 E. 4

29. The first three characters of a bicycle combination lock are letters, followed by three digits, 0 – 9 inclusive. The letters must all be vowels, which cannot repeat, and the digits can repeat. How many bicycle lock combinations are possible?

- A. 125,000 B. 90,000 C. 43,200 D. 60,000 E. 120,000

30. $3221_4 = \underline{\hspace{2cm}}$ (base 9)

- A. 311 B. 302 C. 256 D. 278 E. 262

31. $158^\circ\text{F} = \underline{\hspace{2cm}}^\circ\text{C}$

- A. 80 B. 60 C. 90 D. 100 E. 70

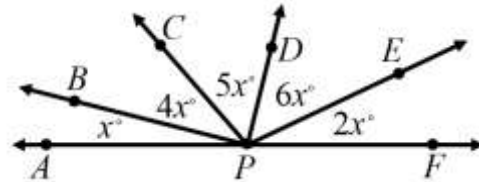
32. $\frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} = \underline{\hspace{2cm}}$

- A. $\frac{5}{32}$ B. $\frac{3}{52}$ C. $\frac{5}{52}$ D. $\frac{7}{18}$ E. $\frac{2}{9}$

33. Humza goes to the store and buys three packs of batteries that cost \$15.00 each, including tax. Humza had a total of \$48.00 before he bought the batteries. What percent of the original \$48.00 will be left after Humza buys the packs of batteries?

- A. 5.25% B. 5.5% C. 5.75% D. 6% E. 6.25%

34. $\angle APF$ is a straight angle. What is the measure of the sum of $\angle BPC$ and $\angle APD$?



- A. 140° B. 120° C. 150° D. 160° E. 170°

35. What are the coordinates of the vertex of the graph of the quadratic equation $y = \frac{1}{4}x^2 - \frac{1}{2}x + \frac{5}{2}$?

- A. $(-1, 2)$ B. $(-1, \frac{1}{2})$ C. $(1, -\frac{3}{4})$ D. $(2, 1\frac{3}{4})$ E. $(1, 2\frac{1}{4})$

36. If $f(x) = x + 3$, $g(x) = 5x$, and $h(x) = 1 - x$, then find $h(g(f(2a)))$.

- A. $-9a - 16$ B. $-10a - 14$ C. $-10a - 9$ D. $-9a + 11$ E. $-10a - 16$

37. $\triangle ABC$ is a 30-60-90 special right triangle. If $\angle C$ is a right angle, and $m\angle A = 60^\circ$, find $\cos(\angle B)$.

- A. $\frac{\sqrt{3}}{2}$ B. $\frac{1}{\sqrt{3}}$ C. $\frac{\sqrt{3}}{1}$ D. $\frac{1}{2}$ E. $\frac{2}{\sqrt{3}}$

38. In the xy -coordinate plane, lines a and b intersect at point $(-10, 5)$, and lines b and c intersect at point $(6, -3)$. What is the slope of any line perpendicular to line b ?

- A. $-\frac{1}{4}$ B. $-\frac{1}{2}$ C. 1 D. 4 E. 2

39. $(\frac{4.05 \times 10^{12}}{5 \times 10^{-4}})(\frac{3.2 \times 10^{-4}}{16 \times 10^2}) = \underline{\hspace{2cm}}$ (scientific notation)

- A. 1.62×10^{10} B. 1.62×10^6 C. 1.62×10^9 D. 8.12×10^8 E. 3.24×10^{10}

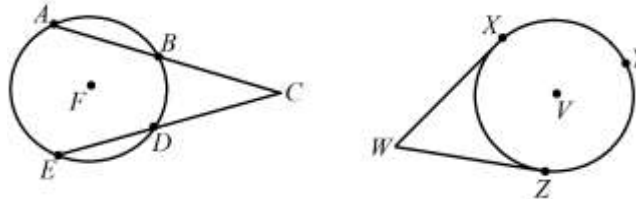
40. If $x^3 \leq 729$, what is the sum of all positive integers, x , that satisfy the inequality?
 A. 36 B. 55 C. 56 D. 45 E. 72

41. Karim’s family went to dinner, which had a subtotal of \$130.00. A 7.5% tax and a 22% tip were added to the bill, both applied to the subtotal. What was the total cost of the dinner, including tax and tip?
 A. \$168.35 B. \$168.85 C. \$166.95 D. \$169.25 E. \$167.65

42. If $\pi = 3$, what is the volume of the sphere with the equation $x^2 + y^2 + z^2 + 6x + 12y + 16z = -45$?
 A. 1,648 units³ B. 2,048 units³ C. 1,864 units³ D. 2,112 units³ E. 2,236 units³

43. What is the sum of the exponents when $\frac{(3a^4bc^4)^2(a^{-5}b^{-1}c^{-3})(a^{10}b^{11}c^{12})}{9abc}$ is simplified?
 A. 42 B. 35 C. 37 D. 41 E. 39

44. In the picture below, if \overline{AC} and \overline{EC} are secants of $\odot F$, the measure of arc $AE = 154^\circ$, the measure of arc $BD = 48^\circ$, \overline{WX} and \overline{WZ} are tangent to $\odot V$, and the measure of arc $XZ = 108^\circ$, then $m\angle W - m\angle C = \underline{\hspace{2cm}}^\circ$.



A. 53 B. 27 C. 38 D. 19 E. 23

45. Simplify: $\frac{6n^2+18n-60}{6n^2-150}$
 A. $\frac{n+2}{n+5}$ B. $\frac{n+5}{n-5}$ C. $\frac{n-2}{n-5}$ D. $\frac{n+2}{n-5}$ E. $\frac{n-2}{n+5}$

46. If the solution to the system of linear equations $\begin{cases} a + b + c = 11 \\ 3a - 2b + c = -10 \\ 2a + 3b + 4c = 45 \end{cases}$ is (a, b, c) , then what is the value of $ab + bc - ac$?
 A. 31 B. 45 C. 81 D. 57 E. 39

47. What is the mean absolute deviation of the set of numbers 59, 47, 55, 46, 48, and 51?
 A. 4 B. 5 C. 4.5 D. 5.5 E. 6

48. Angel and Joy are sisters. Angel can clean their house by herself in 1 hour, and Joy can clean their house by herself in 1.5 hrs. If they work together, how long will it take the sisters to clean their house?
 A. 36 minutes B. 45 minutes C. 54 minutes D. 32 minutes E. 48 minutes

49. If $x + \frac{1}{x} = 4$ and $x^4 + \frac{1}{x^4} = y$, then simplify $\sqrt{y + 6}$.
 A. $\sqrt{262}$ B. $10\sqrt{2}$ C. $16\sqrt{2}$ D. $3\sqrt{43}$ E. $2\sqrt{47}$

50. $\frac{25\pi}{18}$ radians = _____ (degrees)
 A. 230 B. 235 C. 245 D. 240 E. 250

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1. C	18. C	35. E
2. D	19. E	36. B
3. C	20. B	37. A
4. B	21. D	38. E
5. B	22. C	39. C
6. B	23. B	40. D
7. E	24. B	41. A
8. A	25. A	42. B
9. D	26. A	43. E
10. E	27. E	44. D
11. A	28. C	45. C
12. D	29. D	46. D
13. B	30. D	47. A
14. D	31. E	48. A
15. C	32. E	49. B
16. E	33. E	50. E
17. A	34. A	

13. If $m\angle B = 134^\circ$, then its supplement is equal to $180 - 134 = 46^\circ$. If $m\angle A$ is twelve degrees less than the supplement of $\angle B$, then $m\angle A = 46 - 12 = 34^\circ$. The complement of $\angle A$ is then equal to $90 - 34 = 56^\circ$.

22. The pattern of the sequence 0.15, 0.45, 1.35, 4.05, ..., is to multiply the previous term by 3. Therefore, the next term of the sequence is $4.05(3) = 12.15$.

28. Let H = horse, C = cow and CH = chicken. Write the first set of equations from the given information. Next, multiply the first equation by 3. Next, subtract the second equation from the first equation.

$$\begin{array}{r} 1H + 5C = 1CH \\ 3H + 2C = 2CH \end{array} \rightarrow \begin{array}{r} 3(1H + 5C = 1CH) \\ 3H + 2C = 2CH \end{array} \rightarrow \begin{array}{r} 3H + 15C = 3CH \\ 3H + 2C = 2CH \end{array} \rightarrow \begin{array}{r} 3H + 15C = 3CH \\ -3H - 2C = -2CH \\ \hline 13C = 1CH \end{array}$$

After subtracting the second equation from the first equation, 13 cows are equal to 1 chicken.

32. When adding fractions in the form $\frac{1}{a \cdot b} + \frac{1}{b \cdot c} + \frac{1}{c \cdot d}$, the sum is equal to $\frac{\text{sum of numerators}}{a \cdot d}$. The same is true for this problem. Therefore, $\frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} = \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \frac{1}{5 \cdot 6} + \frac{1}{6 \cdot 7} + \frac{1}{7 \cdot 8} + \frac{1}{8 \cdot 9}$, and the sum is equal to $\frac{1+1+1+1+1+1}{3 \cdot 9} = \frac{6}{27} = \frac{2}{9}$.

35. To find the x -coordinate of the vertex of a quadratic equation, use $x = -\frac{B}{2A}$. In the equation quadratic equation $y = \frac{1}{4}x^2 - \frac{1}{2}x + \frac{5}{2}$, $A = \frac{1}{4}$ and $B = -\frac{1}{2}$. The x -coordinate is then $x = \frac{\frac{1}{2}}{2(\frac{1}{4})} = 1$. Substitute 1 into the equation to find the y -coordinate of the vertex, so $y = \frac{1}{4}(1)^2 - \frac{1}{2}(1) + \frac{5}{2} = \frac{9}{4} = 2\frac{1}{4}$. Therefore, the vertex of the quadratic equation has the coordinates $(1, 2\frac{1}{4})$.

36. If $f(x) = x + 3$, $g(x) = 5x$, and $h(x) = 1 - x$, then using order of operations, $f(2a) = 2a + 3$. Next, $g(2a + 3) = 5(2a + 3) = 10a + 15$. Finally, $h(10a + 15) = 1 - (10a + 15) = -10a - 14$.

38. If line b intersects line a at $(-10, 5)$ and line c at $(6, -3)$, then the slope of line b is $\frac{-3-5}{6-(-10)} = \frac{-8}{16} = -\frac{1}{2}$. If a line is perpendicular to line b , then its slope will have a negative reciprocal slope of line b . Therefore, the slope of any line perpendicular to line b is the negative reciprocal slope of $-\frac{1}{2}$, which is 2.

40. Because $9^3 = 729$, then the positive integers that satisfy $x^3 \leq 729$, are 1, 2, 3, 4, 5, 6, 7, 8, and 9. Therefore, the sum of all positive integers, x , that satisfy the inequality is $1 + 2 + 3 + 4 + 5 + 7 + 8 + 9 = 45$.

43. $\frac{(3a^4bc^4)^2(a^{-5}b^{-1}c^{-3})(a^{10}b^{11}c^{12})}{9abc} = \frac{(9a^8b^2c^8)(a^{-5}b^{-1}c^{-3})(a^{10}b^{11}c^{12})}{9abc} = \frac{9a^{8+(-5)+10}b^{2+(-1)+11}c^{8+(-3)+12}}{9abc} = \frac{9a^{13}b^{12}c^{17}}{9abc}$
 $= \frac{9a^{13}b^{12}c^{17}}{9abc} = a^{13-1}b^{12-1}c^{17-1} = a^{12}b^{11}c^{16}$. Therefore, $12 + 11 + 16 = 39$.

45. $\frac{6n^2+18n-60}{6n^2-150} = \frac{6(n^2+3n-10)}{6(n^2-25)} = \frac{6(n-2)(n+5)}{6(n-5)(n+5)} = \frac{6(n-2)(\cancel{n+5})}{6(n-5)(\cancel{n+5})} = \frac{n-2}{n-5}$.