

TMSCA MIDDLE SCHOOL MATHEMATICS<br>TEST \#10 ©<br>FEBRUARY 4, 2017

## GENERAL DIRECTIONS

1. About this test:
A. You will be given 40 minutes to take this test.
B. There are 50 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet be sure to use BLOCK CAPITAL LETTERS. Clean erasures are necessary for accurate grading on Scantrons and Chatsworth cards.
3. If you are using a Chatsworth or Scantron card, please follow the specific instructions given at your particular meet.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have ONE and ONLY ONE correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators MAY NOT be used on this test.
8. All problems answered correctly are worth FIVE points. TWO points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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1. Which expression below produces the smallest sum?
A. $54-360$
B. $700-456$
C. $2-378$
D. 10,000-3
E. $1-300$
2. $\left(4 \frac{1}{2}-6 \frac{2}{3}\right) \cdot\left(7 \frac{3}{4}-10 \frac{1}{2}\right)=$
A. $5 \frac{7}{12}$
B. $5 \frac{23}{24}$
C. $-5 \frac{3}{4}$
D. $-5 \frac{7}{8}$
E. $5 \frac{19}{24}$
3. What is the smallest possible product using three different numbers of the set $\{-9,-6,-3,0,7,8,10\}$ ?
A. -693
B. -540
C. -720
D. 0
E. $-1,620$
4. Michelle is hosting a birthday party for her friend. She bought bunches of flowers. Each bunch has five flowers and $\frac{2}{5}$ of the flowers are yellow. If Michelle bought eleven bunches of flowers, how many are not yellow?
A. 55
B. 22
C. 33
D. 44
E. 11
5. A pair of adjacent dice lie on a table, as in the picture below. What is the product of the faces not shown?

A. 4,320
B. 2,680
C. 3,640
D. 4,840
E. 5,460
6. Mitch and Rosemary are exactly the same height today. In three years, Mitch will have grown $30 \%$ taller, but Rosemary will only have grown one-third as many inches as Mitch. If Mitch and Rosemary are exactly 50 inches tall, how tall will Rosemary be in three years?
A. $4 \frac{7}{12}$ feet
B. $5 \frac{5}{12}$ feet
C. $4 \frac{3}{4}$ feet
D. $4 \frac{1}{6}$ feet
E. $5 \frac{1}{6}$ feet
7. What is the maximum number of intersection points that can be created using only four lines?
A. 4
B. 5
C. 6
D. 7
E. 8
8. Let $A$ equal the sum of -18 and -6 , divided by the product of 2 and -4 . Find the value of $\left(A^{2}+1\right)^{2}$.
A. 100
B. 28.5625
C. 289
D. 9
E. 676
9. What is the prime factorization of $6!$ ?
A. $2^{3}$
B. $2 \cdot 3$
C. $2^{4} \cdot 3^{2} \cdot 5$
D. $2^{3} \cdot 3^{2} \cdot 5^{2}$
E. $2^{4} \cdot 3^{3} \cdot 5$
10. If $A=\{5,10,15,20,25\}$ and $B=\{3,6,9,12,15\}$, then what is the sum of the elements of $A \cup B$ ?
A. 115
B. 120
C. 30
D. 15
E. 105
11. MMDCXLVI - CMXII $=$ $\qquad$ (Roman numeral)
A. MDCCXXXIV
B. MCXLVI
C. MDCLXXVI
D. MDCLXXIV
E. MDCXXXIV
12. $\frac{3.5 \times 10^{13}}{5 \times 10^{7}}=$ $\qquad$
A. $7 \times 10^{6}$
B. $7 \times 10^{5}$
C. $7 \times 10^{4}$
D. $7 \times 10^{7}$
E. $7 \times 10^{8}$
13. Cleavus is buying a shirt that costs $\$ 28$. If the shirt is on sale for $25 \%$ off and there is a $9 \%$ tax, how much will Cleavus have to pay in order to buy the shirt?
A. $\$ 22.29$
B. $\$ 21.79$
C. $\$ 22.69$
D. $\$ 21.39$
E. $\$ 22.89$
14. On a number line, $X$ and $Y$ are located at 16 and 40, respectively. $Z$ is the midpoint of $\overline{X Y}$ and $W$ is the midpoint of $\overline{X Z}$. What is the coordinate of $W$ ?
A. 28
B. 20
C. 22
D. 19
E. 24
15. If $m \angle 8$ is the supplement of a $39.5^{\circ}$ angle and $m \angle 16$ is the complement of a $27.5^{\circ}$, what is the sum of the measures of $\angle 1$ and $\angle 10$ ?

A. $293^{\circ}$
B. $141^{\circ}$
C. $258^{\circ}$
D. $282^{\circ}$
E. $226^{\circ}$
16. Which two shapes below have the exact same area? All measurements are in centimeters.

A. rectangle and triangle B. square and parallelogram C. trapezoid and rectangle D. triangle and square E. trapezoid and triangle
17. $\overline{K L}$ has endpoints $K(-11,17)$ and $L(39,41)$. If point $L$ is translated eight units down and seven units to the left to create point $M$, what is the sum of the coordinates of the midpoint of $\overline{K M}$ ?
A. 50
B. 41.5
C. 43
D. 39.5
E. 35.5
18. What is the product of all the digits of the greatest palindrome less than 9,361 ?
A. 327
B. 729
C. 162
D. 1,296
E. 648
19. Which set of numbers below does not have its mean equal to its median?
A. $\{8,10,12,14\}$
B. $\{3,5,17,19\}$
C. $\{10,11,11,12\}$
D. $\{4,6,16,22\}$
E. $\{1,4,18,21\}$
20. Simplify: $\quad(87 x-19 y+4)-(-11 y+7 x-16)-2(x-y-11)+4 x-3 y-42$
A. $82 x-9 y$
B. $82 x+2 y-32$
C. $100 x-27 y$
D. $100 x-27 y-32$
E. $100 x+82 y$
21. What is the $8^{\text {th }}$ term of the following sequence? $\quad 2.8,8.4,25.2,75.6, \ldots$
A. $4,343.4$
B. 2,041.2
C. $6,123.6$
D. $5,721.6$
E. $6,734.4$
22. A right triangle has legs measuring 12 inches and 16 inches. What is the length of the altitude drawn to the hypotenuse?
A. 20 inches
B. 8.4 inches
C. 9.6 inches
D. 10.4 inches
E. 19.2 inches
23. The sum of four consecutive positive odd integers is 376 . What is the product of the least and greatest of these four integers?
A. 8,835
B. 8,463
C. 9,603
D. 9,215
E. 8,827
24. What is the surface area of a cube whose volume is $512 \mathrm{~cm}^{3}$ ?
A. $192 \mathrm{~cm}^{2}$
B. $384 \mathrm{~cm}^{2}$
C. $768 \mathrm{~cm}^{2}$
D. $576 \mathrm{~cm}^{2}$
E. $460.8 \mathrm{~cm}^{2}$
25. There are 3 red, 3 blue and 3 yellow balls in a bag. Three balls are chosen at random without replacement. What is the probability all three balls chosen are yellow?
A. $\frac{1}{440}$
B. $\frac{1}{3}$
C. $\frac{1}{2}$
D. $\frac{1}{74}$
E. $\frac{1}{84}$
26. $121_{7}+234_{5}=$ $\qquad$
A. 355
B. 354
C. 341
D. 319
E. 345
27. The eight-digit number $108 B 4194$ is a multiple of 11 . What is the value of $B$ ?
A. 5
B. 6
C. 7
D. 8
E. 9
28. If $\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \ldots \cdot \frac{m}{n}=\frac{1}{16}$ and $\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \ldots \cdot \frac{r}{s}=\frac{1}{25}$, then what is the value of $\frac{m}{n} \cdot \frac{r}{s}$ ?
A. $\frac{3}{5}$
B. $\frac{5}{8}$
C. $\frac{9}{16}$
D. $\frac{3}{4}$
E. $\frac{9}{10}$
29. What is the least possible sum of two positive integers whose product is 48 less than twice the square of 12 ?
A. 27
B. 44
C. 40
D. 35
E. 31
30. If $a \uparrow b=\frac{2}{a}+\frac{b}{3}$, then find the value of $((-6 \uparrow(-1)) \uparrow(4 \uparrow 12))$.
A. -4.5
B. -3.5
C. -2.5
D. -1.5
E. -0.5
31. Marta bought seven tickets to a school play for $\$ 17.50$. The price increased the night of the play, so Nathan had to pay $\$ 24.00$ for six tickets. What was the percent of increase in the unit rates for each ticket price?
A. $40 \%$
B. $30 \%$
C. $60 \%$
D. $50 \%$
E. $25 \%$
32. Which of the following linear equations does not have an $x$-intercept of 1 ?
A. $4 x-y=4$
B. $y-3 x=0$
C. $y=-5 x+5$
D. $3.5 y-1.6 x=-2(0.8)$
E. $y-0=3(x-1)$
33. Which solution set below represents the graph?

A. $[82,85)$
B. $(82,85]$
C. $(82,85)$
D. $[82,85]$
E. $[83,84]$
34. If $\frac{1}{x^{2}+4}=0.025$, then $x$ is equal to which of the following?
A. $\{ \pm 6\}$
B. $\{ \pm 9\}$
C. $\left\{ \pm^{2 / 3}\right\}$
D. $\{4\}$
E. $\{ \pm 4\}$
35. Eight identical boxes are placed on the floor such that they are not touching. One box has a length that is two more than its width and its height is five more than its width. What is the combined surface area of all the boxes?
A. $48 w^{2}+24 w+160$
B. $48 w^{2}+224 w+160$
C. $48 w^{2}+224 w+80$
D. $8 w^{2}+24 w+12$
E. $24 w^{2}+66 w$
36. If $f(x, y)=(3 x-2 y)^{2}$, find the value of $-4 f(7,13)$.
A. -100
B. -250
C. $-3,000$
D. 2,500
E. -50
37. A 30-60-90 right triangle is inscribed in a circle of radius 6 cm . What is the length of the longer leg?
A. 6 cm
B. $6 \sqrt{3} \mathrm{~cm}$
C. $12 \sqrt{3}$
D. 12 cm
E. $12 \sqrt{2} \mathrm{~cm}$
38. In base 10 what is the positive difference between the largest three-digit base 6 number and the smallest three-digit base 3 number?
A. 206
B. 155
C. 142
D. 152
E. 154
39. Find the area of a segment of a circle with a central angle of $120^{\circ}$ and a radius of 6 inches.
A. $12 \pi-18 \sqrt{3}$ in $^{2}$
B. $12 \pi-9 \sqrt{3} i n^{2}$
C. $24 \pi-18 \sqrt{3}$ in $^{2}$
D. $12 \pi-9 \sqrt{3} i n^{2}$
E. $12 \pi-6 \sqrt{3} i n^{2}$
40. Find the value of five more than $x$, if $\frac{2}{3} x-(x+0.25)=\frac{1}{12}\left(x+\frac{8}{2}\right)$.
A. -6.4
B. -1.4
C. 3.6
D. 5.8
E. 4.2
41. What is the units of $16^{4}+26^{3}$ ?
A. 2
B. 4
C. 6
D. 8
E. 0
42. Solve for $x . \quad \frac{x^{2}}{x-7}=\frac{49}{x-7}$
A. 7
B. $\pm 7$
C. 3.5
D. -3.5
E. -7
43. What is the area of quadrilateral $A B C D$ that has its vertices located at $A(-9,8), B(7,-7), C(10,-5)$ and $D(14,2)$.
A. 131 units $^{2}$
B. 66 units $^{2}$
C. 132 units $^{2}$
D. 83 units $^{2}$
E. 125 units $^{2}$
44. In the circle below, $B C=24 \mathrm{~cm}$. Find the value of $x$.

A. 16 cm
B. $4 \sqrt{13} \mathrm{~cm}$
C. $(2 \sqrt{2}+4 \sqrt{3}) \mathrm{cm}$
D. $(2 \sqrt{2}+2 \sqrt{3}) \mathrm{cm}$
E. 14 cm
45. In a neighborhood pond, there are 6,000 bullfrogs. The bullfrogs are decreasing at a rate of $40 \%$ per year. How many bullfrogs will be in the neighborhood pond after three years?
A. 384
B. 768
C. 1,646
D. 1,296
E. 3,600
46. Da'vonne was asked to solve the two equations, $-3|4 x-7|=-63$ and $\sqrt{3 x-2}=8$. Once she solved both equations, Da'vonne was asked to sum all the solutions of the equations. What was the sum Da'vonne calculated?
A. 32.5
B. 18.5
C. 25.5
D. 29
E. 32
47. What is the product of the roots of the quadratic equation $y=\frac{1}{4} x^{2}+\frac{5}{8} x+\frac{3}{2}$ ?
A. 2.5
B. -2.5
C. 6
D. $0.41 \overline{6}$
E. 4
48. Simplify: $\left(\frac{9 x^{2} y^{-4}}{3 x y^{-3}}\right)^{-3}$
A. $\frac{y^{3}}{27 x^{3}}$
B. $\frac{y^{3}}{9 x^{3}}$
C. $\frac{27 x^{3}}{y^{3}}$
D. $\frac{9 y^{12}}{x^{3}}$
E. $\frac{x^{3}}{27 y^{3}}$
49. Fernando has a bag consisting of four green marbles and five blue marbles. Fernando must choose three balls at random from his bag. What are the odds all three balls will be blue?
A. 5:42
B. 5:37
C. 5:9
D. 5:15
E. 5:24
50. On a coordinate plane, what is the area of the triangle bounded by the $y$-axis, $x+y=12$ and $y=\frac{5}{2} x-2$ ?
A. 32 units $^{2}$
B. 56 units $^{2}$
C. 34 units $^{2}$
D. 112 units $^{2}$
E. 28 units $^{2}$

| 1. C | 18. B | 35. B |
| :---: | :---: | :---: |
| 2. B | 19. D | 36. A |
| 3. C | 20. A | 37. B |
| 4. C | 21. C | 38. A |
| 5. A | 22. C | 39. B |
| 6. A | 23. E | 40. C |
| 7. C | 24. B | 41. A |
| 8. A | 25. E | 42. E |
| 9. C | 26. C | 43. A |
| 10. E | 27. B | 44. B |
| 11. A | 28. E | 45. D |
| 12. B | 29. E | 46. C |
| 13. E | 30. D | 47. C |
| 14. C | 31. C | 48. A |
| 15. C | 32. B | 49. B |
| 16. C | 33. B | 50. E |
| 17. E | 34. A |  |

12. $\frac{3.5 \times 10^{13}}{5 \times 10^{7}}=\left(\frac{3.5}{5}\right)\left(\frac{10^{13}}{10^{7}}\right)=0.7\left(10^{13-7}\right)=0.7 \times 10^{6}=7 \times 10^{5}$.
13. The volume of a cube is $V=s^{3}$. Since we are given the volume of a cube is 512 , we know that $s=8$, because $8^{3}=512$. To find the surface area of a cube, use $S A=6 s^{2}$. Thus, $6(8)^{2}=384 \mathrm{~cm}^{2}$.
14. If $f(x, y)=(3 x-2 y)^{2}$, then $-4 f(7,13)=-4(3 \cdot 7-2 \cdot 13)^{2}=-4(-5)^{2}=-100$.
15. To find the area of a segment of a circle, you subtract the area of the isosceles triangle formed from the area of the sector of the circle. Draw a picture such as the one below.


The area of the sector is $A=\frac{n}{360} \cdot \pi r^{2}=\frac{120}{360} \cdot 6^{2} \pi=\frac{1}{3} \cdot 36 \pi=12 \pi$. The area of the isosceles triangle can be found drawing a perpendicular bisector to the base of the isosceles triangle and using 30-60-90 right triangle ratios can find the height and base of the isosceles triangle. From the picture above the area of the triangle is $A=\frac{b h}{2}=\frac{3(6 \sqrt{3})}{2}=9 \sqrt{3}$. Thus the area of the segment is $12 \pi-9 \sqrt{3} \mathrm{~cm}^{2}$.
42. From our equation, $\frac{x^{2}}{x-7}=\frac{49}{x-7}$, we see $x^{2}=49$. So, $\sqrt{x^{2}}=\sqrt{49}$ and $x= \pm 7$. However, if you substitute 7 for $x$, we get,,$\frac{7^{2}}{7-7}=\frac{49}{7-7}$, which would give us expressions with zero in the denominator. This is undefined and not possible, so our only solution is $x=-7$.
45. This is an example of an exponential decay problem. The exponential decay form is $y=a \cdot b^{x}$, where $a$ is the initial population, $b$ equals $1-$ rate and $x$ is time. Our equation is $y=6000(1-0.4)^{3}=1,296$.
47. The standard form of a quadratic equation is $y=A x^{2}+B x+C$. The product of the roots is equal to $\frac{C}{A}$. We are given $y=\frac{1}{4} x^{2}+\frac{5}{8} x+\frac{3}{2}$, so the product of the roots is $\frac{3}{2} \div \frac{1}{4}=\frac{3}{2} \cdot \frac{4}{1}=6$.
49. First, find the probability of choosing three blue marbles. Since there are 5 blue marbles and we want 3 , we have ${ }_{5} C_{3}=10$. We have 9 marbles total and choosing three at a time gives us ${ }_{9} C_{3}=84$. Therefore, the probability of getting three blue marbles is $\frac{10}{84}=\frac{5}{42}$. Since odds are the ratio of favorable outcomes to unfavorable outcomes, our odds are 5:37.

