

TMSCA MIDDLE SCHOOL MATHEMATICS<br>TEST \#11 ©<br>FEBRUARY10, 2018

## GENERAL DIRECTIONS

1. About this test:
A. You will be given 40 minutes to take this test.
B. There are 50 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet be sure to use BLOCK CAPITAL LETTERS. Clean erasures are necessary for accurate grading on Scantrons and Chatsworth cards.
3. If you are using a Chatsworth or Scantron card, please follow the specific instructions given at your particular meet.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have ONE and ONLY ONE correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators MAY NOT be used on this test.
8. All problems answered correctly are worth FIVE points. TWO points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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1. $45-123-98=$
A. 70
B. -176
C. 20
D. -172
E. -70
2. $-73 \frac{1}{4}+16 \frac{3}{5}=$ $\qquad$
A. $-56 \frac{3}{20}$
B. $-89 \frac{17}{20}$
C. $-89 \frac{3}{20}$
D. $-57 \frac{11}{20}$
E. $-56 \frac{13}{20}$
3. $1.6 \times 0.4 \times \frac{1}{2}=$
A. 3.2
B. 0.032
C. 32
D. 0.332
E. 0.32
4. $84 \frac{3}{4} \div(-3)=$ $\qquad$
A. 28.24
B. -24.75
C. -28.50
D. -28.75
E. -28.25
5. If Cheyenne can eat an entire cake in 20 minutes, how many cakes can Cheyenne eat in 3 hours?
A. 6 cakes
B. 9 cakes
C. 12 cakes
D 4 cakes
E. 8 cakes

6 . Find the value of $x$.

A. 47
B. 17
C. 27
D. 57
E. 7
7. Billy mowed $5 / 8$ of his yard. What percentage of his yard does Billy still need to mow?
A. $62.5 \%$
B. $87.5 \%$
C. $12.5 \%$
D. $37.5 \%$
E. $24.5 \%$
8. Tristan measured the shadow of his rose bush to be 198 inches long. If he converts this measurement to yards, how many yards long is Tristan's rose bush's shadow?
A. 16.5 yards
B. 5.5 yards
C. 8.25 yards
D. 4.75 yards
E. 5.75 yards
9. If $1,456=2^{a} \cdot 7^{b} \cdot 13^{c}$, then what is the value of $a+2 b+3 c$ ?
A. 6
B. 12
C. 7
D. 11
E. 9
10. Bethany earned $\$ 36,000$ last year. This year, she received a $3 \%$ raise. How much will Bethany make this year?
A. $\$ 1,080$
B. $\$ 37,800$
C. $\$ 37,080$
D. $\$ 37,460$
E. \$37,280
11. What is the value of the mean of all even numbers that can be formed using exactly two of the digits 2,5 and 9 ?
A. $58 . \overline{6}$
B. $74 . \overline{6}$
C. $68 . \overline{3}$
D. 72
E. 70
12. How many positive integral divisors does the number 5,200 have?
A. 36
B. 30
C. 42
D. 34
E. 38
13. Simplify: $\quad \frac{(16-19)^{2}}{5-(-1)} \div \frac{2}{3}$
A. 2.5
B. 2.25
C. 1.75
D. 1
E. 1.5
14. What is the range of the set of data?
$\{189.4,76.7,191.5,65.8,121.8,179.9\}$
A. $137.51 \overline{6}$
B. 127.6
C. 123.6
D. 125.7
E. 114.1
15. Find the value of $20 x$, if $\frac{2}{x}+\frac{2}{x}=8$.
A. 8
B. 12
C. 6
D. 40
E. 10
16. 2,304 ounces $=$ $\qquad$ quarts
A. 72
B. 64
C. 84
D. 48
E. 36
17. In the wild, six adult male lions will eat 150 pounds of meat each day. How many pounds of meat will seventeen adult male lions eat in one day?
A. 375 lbs
B. 400 lbs
C. 425 lbs
D. 475 lbs
E. 525 lbs
18. Mikhel is going to spin the spinner twice. What is the probability the spinner stops on an odd number first and an even number second?

A. $\frac{1}{2}$
B. $\frac{1}{8}$
C. $\frac{15}{32}$
D. $\frac{15}{64}$
E. $\frac{5}{16}$
19. The product of four different positive integers is 100 . What is the sum of these integers?
A. 18
B. 16
C. 20
D. 22
E. 21
20. The table represents the function $F$. Which equation represents the function $F$ ?

| $x$ | 4 | 5 | 10 | 12 |
| :---: | :---: | :---: | :---: | :---: |
| $F(x)$ | 7 | 16 | 91 | 135 |
| 9 | C. $F(x)=2 x-9$ |  |  |  |

D. $F(x)=9 x-2$
E. $F(x)=x^{3}-x^{2}$
21. Alyce is creating her new avatar. She has to choose among 3 different body choices, 4 different hair styles, 5 different shirts and 3 different pairs of pants. How many different avatars could Alyce possibly make, if she must choose one body choice, one hair style, one shirt, one pair of pants and whether or not her avatar will wear glasses?
A. 280
B. 180
C. 360
D. 320
E. 250
22. What is the next term of the sequence? $2 / 3,4,24,144, \ldots$
A. 720
B. 1,008
C. 924
D. 824
E. 864
23. A $3 \times 3 \times 3$ cube has all of its faces painted blue. If the cube is separated into 27 unit cubes, how many of the unit cubes have exactly three of its faces painted blue?
A. 12
B. 8
C. 10
D. 6
E. 4
24. An alien space ship can travel $1.4 \times 10^{5}$ miles per day. How long can the alien space ship travel in 15 days?
A. $2.1 \times 10^{6}$ miles
B. $2.9 \times 10^{6}$ miles
C. $21 \times 10^{6}$ miles
D. $2.1 \times 10^{7}$ miles
E. $2.9 \times 10^{7}$ miles
25. The average age of the eight people in room A is 25 and the average age of the twelve people in room B is 50 . If the people in the two rooms are put into one room, what is the average age of all twenty people?
A. 32
B. 34
C. 36
D 38
E. 40
26. At the restaurant Underwater Adventure, there is an aquarium with various exotic fish. The aquarium is $\frac{5}{6}$ full. After 30 gallons of water is removed from the aquarium, the aquarium is $\frac{3}{4}$ full. When full, how much water does the aquarium hold?
A. 720 gallons
B. 900 gallons
C. 450 gallons
D. 360 gallons
E. 270 gallons
27. If $A=-5 n-1, B=14 n-6$ and $C=8 n+3$, find $A+B+C$.
A. $27 n-4$
B. $17 n-4$
C. $9 n-7$
D. $17 n-7$
E. $22 n+4$
28. If the side lengths of a triangle are 9,12 and 17 units long, then the triangle is which of the following?
A. right
B. isosceles
C. equilateral
D. acute
E. obtuse
29. There are 124 pigs on a farm. Which exponential function models the population of pigs increasing at a rate of $2 \%$ each year for 9 years?
A. $y=124(9)^{2}$
B. $y=124(2)^{9}$
C. $y=124(1.02)^{9}$
D. $y=124(0.02)^{x}$
E. $y=124(0.98)^{x}$
30. Using the magic square below, find the value of $2 b^{3}+d e-c+2 a$.

| 11 | $a$ | 3 |
| :---: | :---: | :---: |
| $b$ | 9 | $c$ |
| 15 | $d$ | $e$ |

A. 41
B. 54
C. 38
D. 50
E. 46
31. $321_{4}-14_{8}=$ $\qquad$ (base 10)
A. 37
B. 293
C. 307
D. 43
E. 45
32. Simplify: $\frac{\left(3 m^{2} n^{-2}\right)^{2}}{3\left(3 m^{-1} n^{-3}\right)^{2}}$
A. $\frac{3 m^{6}}{n^{2}}$
B. $\frac{3 n^{2}}{m^{6}}$
C. $\frac{m^{6}}{3 n^{2}}$
D. $\frac{n^{2}}{3 m^{6}}$
E. $\frac{m^{6} n^{2}}{3}$
33. What is the value of $x$, if the first three terms of an arithmetic sequence are $4 x-7,3 x+1$ and $5 x-9$, in that order?
A. 19
B. 17
C. 6
D. 4
E. 7
34. A right triangle has legs $a$ and $b$ with hypotenuse $c$. A primitive Pythagorean triple is one in which $a, b$ and $c$ are all relatively prime to each other. Which of the following is not a primitive Pythagorean triple?
A. $6,8,10$
B. 5, 12, 13
C. $3,4,5$
D. $8,15,17$
E. $12,35,37$
35. A convex hexagon has interior angles measuring $(6 x-3)^{\circ},(11 x+1)^{\circ},(10 x-7)^{\circ},(8 x+2)^{\circ},(9 x+15)^{\circ}$ and $(6 x+12)^{\circ}$. What is the measure of the smallest angle?
A. $75^{\circ}$
B. $93^{\circ}$
C. $81^{\circ}$
D. $87^{\circ}$
E. $79^{\circ}$
36. Mark's eco-friendly car can travel 32 miles on one gallon of gasoline. If gas currently costs $\$ 2.25$, how many miles can Mark drive on $\$ 18.00$ worth of gas?
A. 252 miles
B. 256 miles
C. 248 miles
D. 288 miles
E. 224 miles
37. What is the units digit of $8^{10}$ ?
A. 2
B. 4
C. 6
D. 0
E. 8
38. Aneesha is at Jump, Jump, Jump Bounce House. She is standing at the top of a ride and must jump and try to land in the shaded region of the rectangle below. What is the probability Aneesha will land in the shaded region?

A. $16 \%$
B. $64 \%$
C. $78 \%$
D. $92 \%$
E. $84 \%$
39. The top of one flag pole is twelve feet higher than the top of another flag pole. The ratio of the heights of the flag poles is $3: 4$. How many feet high is the taller flag pole?
A. 36 feet
B. 48 feet
C. 52 feet
D. 46 feet
E. 40 feet
40. The formula for the area of a rhombus is $A=\frac{d_{1} d_{2}}{2}$. The first diagonal, $d_{1}$, can be expressed as which of the following?
A. $\frac{A d_{2}}{2}$
B. $\frac{2 d_{2}}{A}$
C. $\frac{2 A}{d_{2}}$
D. $\frac{A}{2 d_{2}}$
E. $2 A d_{2}$
41. A line with an equation of $3 x-6 y=8$ is perpendicular to a line with an equation of $4 x+B y=7$. What is the value of $B$ ?
A. 2
B. -1
C. $-1 / 2$
D. 8
E. -8
42. The tire of a car has a radius of 12 inches. How many revolutions does the tire need to make for the car to travel 10,800 inches? Let $\pi=3$.

A. 150
B. 98
C. 220
D. 180
E. 120
43. What are the roots of the quadratic equation $2 x^{2}-16=12 x$ ?
A. $3 \pm \sqrt{17}$
B. $-3 \pm \sqrt{17}$
C. $-3-\sqrt{17}$
D. $3+\sqrt{17}$
E. $3-\sqrt{17}$
44. A regular hexagon has a side length of 4 cm and an apothem of $2 \sqrt{3} \mathrm{~cm}$. What is the area of the hexagon?
A. $16 \sqrt{3} \mathrm{~cm}^{2}$
B. $8 \sqrt{3} \mathrm{~cm}^{2}$
C. $4 \sqrt{3} \mathrm{~cm}^{2}$
D. $24 \sqrt{3} \mathrm{~cm}^{2}$
E. $20 \sqrt{3} \mathrm{~cm}^{2}$
45. Solve for $n$ :

$$
|3 n-2|=2 n-1
$$

A. $\{1\}$
B. $\{-1,1\}$
C. $\left\{-1,-\frac{3}{5}\right\}$
D. $\left\{\frac{3}{5}, 1\right\}$
E. $\left\{-1, \frac{3}{5}\right\}$
46. Simplify: $\quad \frac{15 a^{2}+7 a b-2 b^{2}}{5 a-b}$
A. $4 a-b$
B. $2 a-3 b$
C. $3 a-2 b$
D. $2 a+3 b$
E. $3 a+2 b$
47. If $x+y=7$ and $x y=9$, what is the value of $x^{2}+y^{2}$ ?
A. 27
B. 31
C. 24
D. 33
E. 29
48. Soaked and Soggy Water Park has 16 different water rides to choose from. In how many ways can a person choose to ride 12 rides to ride during a visit to Soaked and Soggy Water Park?
A. 3,360
B. 1,680
C. 1,820
D. 1,440
E. 1,560
49. A $40 \%$ acid solution is to be mixed with a $70 \%$ acid solution to get 120 liters of a $50 \%$ acid solution. How many liters of the $40 \%$ acid solution are needed?
A. 80 liters
B. 40 liters
C. 60 liters
D. 50 liters
E. 70 liters
50. $\left(x^{16}+1\right)\left(x^{8}+1\right)\left(x^{4}+1\right)\left(x^{2}+1\right)(x+1)(x-1)=$ $\qquad$
A. $x^{32}+1$
B. $x^{32}-1$
C. $x^{128}+1$
D. $x^{128}-1$
E. $x^{64}-1$

| 1. B | 18. D | 35. C |
| :---: | :---: | :---: |
| 2. E | 19. A | 36. B |
| 3. E | 20. B | 37. B |
| 4. E | 21. C | 38. E |
| 5. B | 22. E | 39. B |
| 6. A | 23. B | 40. C |
| 7. D | 24. A | 41. A |
| 8. B | 25. E | 42. A |
| 9. E | 26. D | 43. A |
| 10. C | 27. B | 44. D |
| 11. D | 28. E | 45. D |
| 12. B | 29. C | 46. E |
| 13. B | 30. E | 47. B |
| 14. D | 31. E | 48. C |
| 15. E | 32. E | 49. A |
| 16. A | 33. C | 50. B |
| 17. C | 34. A |  |

13. $\frac{(16-19)^{2}}{5-(-1)} \div \frac{2}{3}=\frac{(-3)^{2}}{6} \div \frac{2}{3}=\frac{9}{6} \div \frac{2}{3}=\frac{9}{6} \cdot \frac{3}{2}=\frac{27}{12}=\frac{9}{4}=2.25$.
14. Adding fractions, $\frac{2}{x}+\frac{2}{x}=\frac{4}{x}$. So, now we have the equation $\frac{4}{x}=8$. Change 8 to $\frac{8}{1}$ and make the proportion $\frac{4}{x}=\frac{8}{1}$. Cross multiply and $4=8 x$. Divide both sides by 8 and $x=1 / 2$. Therefore, $20(1 / 2)=10$.
15. There are 32 ounces in 1 quart, so $2,304 \div 32=72$ quarts.
16. Alyce has to choose among 3 different body choices, 4 different hair styles, 5 different shirts, 3 different pairs of pants and whether her avatar will wear glass or not, so $3 \cdot 4 \cdot 5 \cdot 3 \cdot 2=360$ different avatars Alyce could possibly create.
17. Set up the equation $\frac{5}{6} x-30=\frac{3}{4} x$, with $x$ equal to the total gallons of the aquarium. Start by multiplying both sides of the equation by 12 , to eliminate the fractions, $12\left(\frac{5}{6} x-30=\frac{3}{4} x\right) \rightarrow 10 x-360=9 x$. Add 360 to both sides and subtract $9 x$ to both sides and get $x=360$ gallons.
$36 . \$ 18 \div \$ 2.25=8$ gallons. $8 \times 32=256$ total miles Mark can drive on $\$ 18$ worth of gas.
18. We see a pattern as, $8^{1}=8,8^{2}=64,8^{3}=512$ and $8^{4}=4,096$, with the units digit being $8,4,2$ and 6 . This pattern of units digit repeats. Divide 10 by 4 and get a remainder of 2 , which makes us look at $8^{2}$. So, $8^{10}$ will have the same units digit as $8^{2}$, which has a units digit of 4 .
19. The area of the rectangle is $15 \times 20=300 \mathrm{ft}^{2}$. The area of the triangle is $\frac{8 \times 12}{2}=48 \mathrm{ft}^{2}$. The probability Aneesha will land in the shaded region is then $\frac{300-48}{300}=\frac{252}{300}=\frac{21}{25}=0.84=84 \%$.
20. Set up the proportion $\frac{3}{4}=\frac{x}{x+12}$, with $x$ equal to the shorter pole height. Cross multiply and we get $3(x+12)=4 x$ and then distribute to get $3 x+36=4 x$. Subtract $3 x$ from both sides and $x=36$. The height of the taller flag pole is then $36+12=48$ feet.
21. The formula to find the area of any regular polygon when given the apothem is $A=\frac{a P}{2}$, where $a=$ apothem and $P=$ perimeter. We are given a hexagon with a side length of 4 cm and an apothem of $2 \sqrt{3} \mathrm{~cm}$. Substituting into the formula and we get $A=\frac{a P}{2}=\frac{(2 \sqrt{3})(6 \cdot 4)}{2}=\frac{(2 \sqrt{3})(24)}{2}=\frac{48 \sqrt{3}}{2}=24 \sqrt{3} \mathrm{~cm}^{2}$.
22. To simplify $\frac{15 a^{2}+7 a b-2 b^{2}}{5 a-b}$, first factor the numerator, $\frac{15 a^{2}+7 a b-2 b^{2}}{5 a-b}=\frac{(3 a+2 b)(5 a-b)}{5 a-b}$. Now, simplify the rational expression and $\frac{(3 a+2 b)(5 a-b)}{5 a-b}=3 a+2 b$.
23. We are given that $x+y=7$. If we square both sides, we get $(x+y)^{2}=7^{2}$ and then $x^{2}+2 x y+y^{2}=49$. We are told that $x y=9$, so we can substitute and $x^{2}+2(9)+y^{2}=49$, which gives us $x^{2}+y^{2}+18=49$. Now, subtract 18 from both sides and $x^{2}+y^{2}=49-18=31$.
24. We are given $\left(x^{16}+1\right)\left(x^{8}+1\right)\left(x^{4}+1\right)\left(x^{2}+1\right)(x+1)(x-1) .(x+1)(x-1)=x^{2}-1$. Now we have $\left(x^{16}+1\right)\left(x^{8}+1\right)\left(x^{4}+1\right)\left(x^{2}+1\right)\left(x^{2}-1\right) .\left(x^{2}+1\right)\left(x^{2}-1\right)=\left(x^{4}-1\right)$. Now we have $\left(x^{16}+1\right)\left(x^{8}+1\right)\left(x^{4}+1\right)\left(x^{4}-1\right) .\left(x^{4}+1\right)\left(x^{4}-1\right)=\left(x^{8}-1\right)$. Now we have $\left(x^{16}+1\right)\left(x^{8}+1\right)\left(x^{8}-1\right)$. $\left(x^{8}+1\right)\left(x^{8}-1\right)=\left(x^{16}-1\right)$. Now we have $\left(x^{16}+1\right)\left(x^{16}-1\right)$, and $\left(x^{16}+1\right)\left(x^{16}-1\right)=x^{32}-1$.
