TMSCA MIDDLE SCHOOL MATHEMATICS TEST #12 ©

FEBRUARY 17, 2018

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 50 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading on Scantrons and Chatsworth cards.
3. If you are using a Chatsworth or Scantron card, please follow the specific instructions given at your particular meet.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators **MAY NOT** be used on this test.
8. All problems answered correctly are worth **FIVE** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2017 – 2018 TMSCA Middle School Mathematics Test #12

1. $3\frac{1}{2} + 8\frac{1}{4} + 10\frac{1}{8}$ = _____

A. $21\frac{3}{8}$ B. $21\frac{3}{14}$ C. $21\frac{7}{8}$ D. $21\frac{5}{14}$ E. $21\frac{3}{4}$

2. $-24\frac{1}{6} - 8\frac{3}{4}$ = _____

A. $-32\frac{1}{3}$ B. $-32\frac{2}{3}$ C. $-16\frac{2}{3}$ D. $-32\frac{11}{12}$ E. $-16\frac{11}{12}$

3. $848 \div 0.04 \div 100$ = _____

A. 2.12 B. 0.212 C. 212 D. 21.2 E. 2,120

4. $5\frac{3}{4} \times 12.5$ = _____ (nearest whole number)

A. 72 B. 71 C. 70 D. 71.8 E. 71.9

5. Shahem is 2.4 meters tall, Halik is 1.9 meters tall and Sheeza is 2.1 meters tall. How many total centimeters is their combined height?

A. 6.4 cm B. 64 cm C. 640 cm D. 6,400 cm E. 644 cm

6. 3.4×10^{-5} = _____

A. 340,000 B. 340.000 C. 0.00034 D. 0.000034 E. 0.0000034

7. There are seven people in a room. If every person shakes hands with every other person only once, how many total handshakes will take place?

A. 19 B. 20 C. 21 D. 22 E. 23

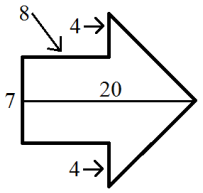
8. $1{,}234 \cdot 9 + 5$ = _____

A. 11,101 B. 11,111 C. 11,311 D. 11,911 E. 11,991

9. What is the sum of the distinct prime factors of 580?

A. 36 B. 38 C. 28 D. 33 E. 34

10. What is the area of the shape below?



A. 112 units^2 B. 152 units^2 C. 122 units^2 D. 146 units^2 E. 90 units^2

11. 48 quarters + 32 nickels = 51 dimes + _____ nickels

A. 170 B. 140 C. 190 D. 210 E. 205

12. 18 is 4% of what number?

A. 45 B. 0.45 C. 4,500 D. 4.5 E. 450

13. Simplify: $8\left(\frac{-2}{6-8} + \frac{(1+2+3+4+5)^0}{8}\right)$

A. 8 B. 7 C. 6 D. 4 E. 9

14. What is the remainder when 35,812 is divided by the number 7?

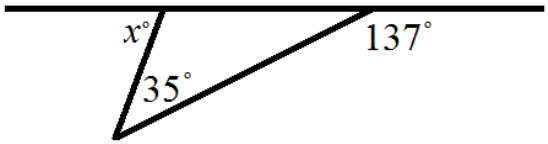
A. 4 B. 3 C. 2 D. 1 E. 0

15. What is the 6^{th} term of the sequence? 2, 10, 50, 250, …
A. 1,250 B. 2,500 C. 12,250 D. 6,250 E. 9,375

16. *TMSCA Math-nuggets* are packed in boxes of 7 or in boxes of 12. What is the smallest number of full boxes needed to pack exactly 2,018 *TMSCA Math-nuggets*.
A. 154 B. 157 C. 169 D. 171 E. 163

17. The complement of $\angle A$ is one-fourth the measure of $\angle A$. What is the measure of $\angle A$?
A. 18° B. 64° C. 58° D. 72° E. 88°

18. Using the picture below, what is the value of x?



A. 82 B. 102 C. 78 D. 43 E. 66

19. It takes 3 teaspoons of seasoning to season 8 fillets of fish. How many teaspoons of seasoning are needed to season four-dozen fish fillets?
A. 18 B. 24 C. 16 D. 20 E. 32

20. If $A = 1$, $B = 2$, $C = 3$, …, $Y = 25$ and $Z = 26$, what is the sum of the letters of the word *EQUAL*?
A. 63 B. 64 C. 52 D. 58 E. 56

21. It is now 10:30 am. What time will it be 500 minutes from now?
A. 5:40 pm B. 5:50 pm C. 6:20 pm D. 6:50 pm E. 7:10 pm

22. Which of the following below is true?
A. $\frac{1}{2} > \frac{3}{4}$ B. $\frac{2}{3} > \frac{1}{4}$ C. $\frac{1}{3} > \frac{1}{2}$ D. $\frac{7}{8} < \frac{3}{4}$ E. $1\frac{1}{2} > 2$

23. How many numbers less than 40 are relatively prime to 40?
A. 20 B. 18 C. 16 D. 14 E. 12

24. For her swimming workout, Swetha needs to swim 18 laps in the pool. Each lap is 60 yards long. So far, Swetha has swum 432 yards. What percentage of her swim workout does Swetha still need to complete?
A. 55% B. 60% C. 65% D. 62.5% E. 67.5%

25. How many total diagonals can be drawn inside of a regular nonagon?
A. 32 B. 45 C. 27 D. 18 E. 9

26. What is the sum of the LCM of 12 and 54 and the GCF of 72 and 108?
A. 144 B. 64 C. 246 D. 224 E. 176

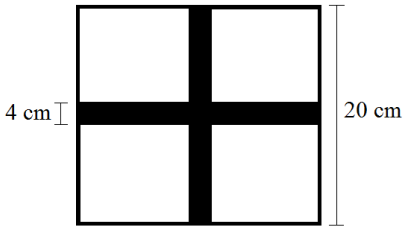
27. Jamie is redecorating her kitchen back splash. She wants to use 6 inch × 6 inch tiles to cover a section of her wall that is 3 feet × 6 feet. How many tiles will Jamie need?
A. 84 tiles B. 108 tiles C. 81 tiles D. 70 tiles E. 72 tiles

28. It takes Johnny 18 minutes jogging at an average rate of 6 mph to get to school. How many minutes would it take Johnny to get to school if he walked at an average rate of 4 mph?
A. 35 minutes B. 54 minutes C. 30 minutes D. 27 minutes E. 24 minutes

29. You flip a coin four times. What is the probability you will get at least two heads?

A. $\frac{1}{4}$ B. $\frac{5}{16}$ C. $\frac{5}{8}$ D. $\frac{3}{8}$ E. $\frac{11}{16}$

30. A dart is thrown at the square dartboard below. What is the probability the dart lands in the shaded region?



A. 36% B. 45% C. 24% D. 28% E. 40%

31. Find the value of (4 ▲ 6) ▲ (8 ▲ 2), if a ▲ $b = \frac{-b}{a-b}$.

A. $-\frac{1}{3}$ B. $\frac{1}{10}$ C. $-\frac{1}{6}$ D. $\frac{1}{12}$ E. $\frac{1}{2}$

32. $412_6 - 32_9$ = _____ (base 7)

A. 243 B. 234 C. 324 D. 240 E. 235

33. Let A be equal to the growth factor of the exponential function $y = 98(4.5)^x$. What is the value of $A^2 + 40A$?

A. 200.50 B. 200.25 C. 200.75 D. 152.25 E. 200.00

34. Simplify: $\frac{(5x^3z^2)^3}{(x^3y^{-4}z^2)(x^{-4}z^3)}$

A. $125x^{10}y^4z$ B. $15x^{10}y^4z$ C. $125x^3y^4z$ D. $125x^{10}y^4z^2$ E. $15x^{10}y^4z^2$

35. The digits of Lucinda's age are interchanged and then 6 is subtracted. The result is one-half of Lucinda's current age. What is Lucinda's current age?

A. 84 B. 62 C. 68 D. 66 E. 86

36. In the addition problem below, each letter represents different non-zero digits and identical letters represent the same digit. What of the following could be a value for E?

$$\begin{array}{r} \text{S A T} \\ \text{S A T} \\ +\ \text{S A T} \\ \hline \text{R E S T} \end{array}$$

A. 4 B. 2 C. 6 D. 7 E. 3

37. What is the slope of a line that passes through the points $(a + 1, 4a)$ and $(a - 1, 2a)$?

A. $\frac{1}{a}$ B. undefined C. 0 D. $\frac{1}{a+1}$ E. a

38. If $f(x) = 5x^2$, then find the value of $f(x + 1)$.

A. $25x^2 + 50x + 25$ B. $10x + 10$ C. $10x^2 + 10x + 10$ D. $5x^2 + 10x + 5$ E. $5x^2 + 5x + 5$

39. Simplify: $\sqrt{\sqrt{625}}$

A. 15 B. $\frac{1}{5}$ C. 5 D. 25 E. $5\sqrt{5}$

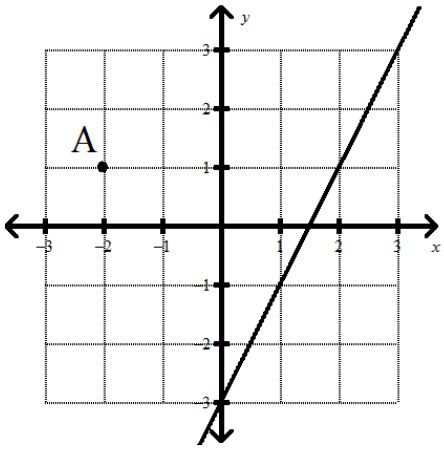
40. Point *A* has coordinates (5, 8) and is rotated 90° counterclockwise. What are point *A*'s new coordinates?
A. $(-5, 8)$ B. $(-8, 5)$ C. $(-5, -8)$ D. $(-8, -5)$ E. $(5, -8)$

41. Two buildings are 16 meters apart. The buildings are 40 meters tall and 70 meters tall, respectively. A high wire will attach the two buildings at their roofs. How long does the wire need to be?
A. 28 meters B. 30 meters C. 34 meters D. 37 meters E. 38 meters

42. Factor completely: $77x^2 - 124x + 20$
A. $(11x - 2)(7x - 10)$ B. $(11x - 10)(7x - 2)$ C. $(11x - 2)(7x + 10)$ D. $(11x + 2)(7x + 10)$ E. $(11x + 2)(7x - 10)$

43. What is the area of the trapezoid bounded by the graphs of the equations $x = 6$, $y = x$, $y = 4$ and the *x*-axis?
A. 16 units^2 B. 24 units^2 C. 32 units^2 D. 12 units^2 E. 18 units^2

44. $\overleftrightarrow{AB}$ is parallel to the line graphed below. Which of the following could be the coordinates of point *B*?



A. (1, -1) B. (5, 16) C. (6, 9) D. (4, 13) E. (2, 8)

45. A cone has a height of 6 inches and a base circle with a diameter of 10 inches. If $\pi = 3.1$, what is the volume of the cone, to the nearest whole number?
A. 125 in^3 B. 155 in^3 C. 124 in^3 D. 150 in^3 E. 175 in^3

46. Scott drives by a pig and chicken farm and sees a total of 47 heads and 116 legs. How many more chickens than pigs did Scott see?
A. 32 B. 18 C. 25 D. 41 E. 22

47. Marcel determines the zeros of the function $f(x)$ to be -8 and 5. Which of the following could be Marcel's function?
A. $f(x) = x^2 + 3x - 40$ B. $f(x) = x^2 - 3x - 40$ C. $f(x) = x^2 + 13x - 40$ D. $f(x) = x^2 - 13x - 40$ E. $f(x) = x^2 + 3x + 40$

48. Simplify: $4\left(\sqrt{20} + 3\sqrt{80}\right) + \sqrt{45}$
A. $27\sqrt{5}$ B. $59\sqrt{5}$ C. $30\sqrt{5}$ D. $63\sqrt{5}$ E. $43\sqrt{5}$

49. There are 11 players on the *Little Strikers* soccer team. If only 7 of the 11 players can be on the field at the same time, how many different combinations of players can be selected to play on the field at one time?
A. 1,663,200 B. 330 C. 77 D. 154 E. 308

50. What is the solution set of the inequality $|w - 3| < 4$?
A. $w < 7$ B. $w < -7$ C. $w > -1$ D. $w > -1$ or $w < 7$ E. $-1 < w < 7$

2017 – 2018 TMSCA Middle School Mathematics Test #12 Answer Key

1. C
2. D
3. C
4. A
5. C
6. D
7. C
8. B
9. A
10. D
11. A
12. E
13. E
14. E
15. D
16. C
17. D
18. C
19. A
20. E
21. D
22. B
23. C
24. B
25. C
26. A
27. E
28. D
29. E
30. A
31. B
32. B
33. B
34. A
35. A
36. C
37. E
38. D
39. C
40. B
41. C
42. A
43. A
44. D
45. B
46. C
47. A
48. B
49. B
50. E

2017-2018 TMSCA Middle School Mathematics Test #12 Selected Answers

7. There are seven people in a room, we will call them *A* – *G*. *A* shakes hands with the other 6 people. *B* has already shaken *A*'s hand, so they will only shake hands with the remaining 5 people. *C* has already shaken hands with *A* and *B*, so they will only shake hands with the remaining 4 people. This pattern continues until we are left with only 1 handshake between *F* and *G*. Therefore, we will have a total of 6 + 5 + 4 + 3 + 2 + 1 = 21 handshakes.

12. Create the equation 18 = 0.04*x*. Divide both sides by 0.04 and $x = \frac{18}{0.04} = 450$.

16. $2018 \div 12 = 168$ with a remainder of 2. If 168 boxes are used, there will be a remainder of 2, which is not a multiple of 7. S, if 167 boxes are used there will be a remainder of 14, which is a multiple of 7 because $14 \div 7 = 2$. Therefore, 167 boxes of 12 and 2 boxes of seven will be needed which is a total of 167 + 2 = 169 boxes.

29. If you flip a coin 4 times, there are 16 possible outcomes: HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, HTTH, HTTT, THTT, TTHT, TTTH, TTHH, THTH, THHT and TTTT. To have at least two heads, then we want HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, THHT, TTHH, THTH and THHT, which is $\frac{11}{16}$.

31. We want to simplify (4 ▲ 6) ▲ (8 ▲ 2), if $a \blacktriangle b = \frac{-b}{a-b}$. First, simplify (4 ▲ 6). If $a \blacktriangle b = \frac{-b}{a-b}$, then (4 ▲ 6) = $\frac{-6}{4-6} = \frac{-6}{-2} = 3$. Now simplify (8 ▲ 2). If $a \blacktriangle b = \frac{-b}{a-b}$, then (8 ▲ 2) = $\frac{-2}{8-2} = \frac{-2}{6} = -\frac{1}{3}$. Now we must simplify $3 \blacktriangle -\frac{1}{3}$. If $a \blacktriangle b = \frac{-b}{a-b}$, then $3 \blacktriangle -\frac{1}{3} = \frac{\frac{1}{3}}{3-\left(-\frac{1}{3}\right)} = \frac{\frac{1}{3}}{3+\frac{1}{3}} = \frac{\frac{1}{3}}{3\frac{1}{3}} = \frac{\frac{1}{3}}{\frac{10}{3}} = \frac{1}{3} \div \frac{10}{3} = \frac{1}{3} \cdot \frac{3}{10} = \frac{3}{30} = \frac{1}{10}$.

34. $\frac{\left(5x^3z^2\right)^3}{(x^3y^{-4}z^2)(x^{-4}z^3)} = \frac{125x^9z^6}{x^{-1}y^{-4}z^5} = 125x^{9-(-1)}y^4z^{6-5} = 125x^{10}y^4z^1 = 125x^{10}y^4z$.

37. The slope formula of two points (x_1, y_1) and (x_2, y_2) is $\frac{y_2-y_1}{x_2-x_1}$. We are given the points (*a* + 1, 4*a*) and $(a-1, 2a)$. So, substitute into the formula and $\frac{4a-2a}{(a+1)-(a-1)} = \frac{2a}{a+1-a+1} = \frac{2a}{2} = a$. The slope of the line that passes through the points (*a* + 1, 4*a*) and $(a-1, 2a)$ is *a*.

38. If $f(x) = 5x^2$, then $f(x+1) = 5(x+1)^2 = 5(x^2+2x+1) = 5x^2+10x+5$.

40. If a point (*x*, *y*) is rotated 90° counterclockwise, then the new coordinates are $(-y, x)$. We are given the point (5, 8), so after a rotation of 90° counterclockwise, then the point's new coordinates are $(-8, 5)$.

46. Let *P* = pigs and *C* = chickens. If there are 47 heads and 116 legs, we create the system $\begin{cases} P + C = 47 \\ 4P + 2C = 116 \end{cases}$. One method to solve this system is to use elimination. We will eliminate the *C* first, s multiply the first equation by -2. We now have $\begin{cases} -2P - 2C = -94 \\ 4P + 2C = 116 \end{cases}$. Add the two equations together and we are left with 2*P* = 22. Divide both sides by 2 and we get *P* = 11. Using the first equation, if *P* = 11, then we have 11 + *C* = 47. Subtract 11 from both sides and *C* = 36. We want to know how many more chickens there are than pigs, so 36 – 11 = 25. There are 25 more chickens than pigs.

48. $4\left(\sqrt{20} + 3\sqrt{80}\right) + \sqrt{45} = 4\left(\sqrt{4 \cdot 5} + 3\sqrt{16 \cdot 5}\right) + \sqrt{9 \cdot 5} = 4\left(2\sqrt{5} + 3 \cdot 4\sqrt{5}\right) + 3\sqrt{5} = 4\left(2\sqrt{5} + 12\sqrt{5}\right) + 3\sqrt{5} = 4\left(14\sqrt{5}\right) + 3\sqrt{5} = 56\sqrt{5} + 3\sqrt{5} = 59\sqrt{5}$.

50. Since this is an absolute value inequality $|w-3| < 4$, it must be rewritten as $w - 3 < 4$ and $w - 3 > -4$. To solve both inequalities, add three to both sides and we get $w < 7$ and $w > -1$. These solutions can be written as one compound inequality as $-1 < w < 7$.