

# TMSCA MIDDLE SCHOOL MATHEMATICS <br> TEST \#13 © <br> FEBRUARY24, 2018 

## GENERAL DIRECTIONS

1. About this test:
A. You will be given 40 minutes to take this test.
B. There are 50 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet be sure to use BLOCK CAPITAL LETTERS. Clean erasures are necessary for accurate grading on Scantrons and Chatsworth cards.
3. If you are using a Chatsworth or Scantron card, please follow the specific instructions given at your particular meet.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have ONE and ONLY ONE correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators MAY NOT be used on this test.
8. All problems answered correctly are worth FIVE points. TWO points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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1. $-98+(-118)+376=$ $\qquad$
A. 180
B. 160
C. -180
D. -592
E. 396
2. $6,000-7,124-(-349)=$ $\qquad$
A. 13,473
B. -775
C. $-1,473$
D. $-1,475$
E. 2,366
3. $24 \frac{3}{4} \div 2.5=$ $\qquad$
A. 9.5
B. 9.6
C. 9.9
D. 10.1
E. 10.7
4. $124 \times \frac{3}{2} \times 1.5=$ $\qquad$
A. 279
B. 186
C. $55 . \overline{1}$
D. 558
E. 324
5. The nucleus of a human cell is about 0.000007 meters in diameter. What is the radius of the nucleus of a human cell?
A. $3.5 \times 10^{-6}$
B. $3.25 \times 10^{-6}$
C. $7 \times 10^{-6}$
D. $1.4 \times 10^{-6}$
E. $1.4 \times 10^{-7}$
6. What is the GCF of the numbers 360 and 48 ?
A. 18
B. 16
C. 24
D. 12
E. 8
7. What is the sum of the digits of the sum of $1,299+785$ ?
A. 21
B. 20
C. 17
D. 16
E. 14
$8.3 / 4 \%$ of $120=3 \%$ of what number?
A. 30
B. 60
C. 15
D. 45
E. 12
8. On Maya's test, one question asked how many inches were in one mile. Maya incorrectly answered 63,110 inches. How many more inches should Maya have added to her answer in order for her answer to have been correct?
A. 250 inches
B. 220 inches
C. 240 inches
D. 300 inches
E. 270 inches
9. A baseball diamond's home plate is in the shape of a pentagon. The home plate has three right angles and the remaining two angles are congruent. What is the measure of one of the non-right angles?
A. $120^{\circ}$
B. $150^{\circ}$
C. $135^{\circ}$
D. $145^{\circ}$
E. $105^{\circ}$
10. The table represents the function $F$. Which equation represents the function $F$ ?

| $x$ | -3 | 0 | 6 | 15 |
| :---: | :---: | :---: | :---: | :---: |
| $F(x)$ | 6 | -3 | 33 | 222 |

A. $F(x)=4 x+18$
B. $F(x)=x^{2}-3$
C. $F(x)=-2 x$
D. $F(x)=\frac{-18}{x}$
E. $f(x)=3 x^{2}+7 x$
12. $45,652 \div 11=4,150$ with a remainder of $\qquad$ _.
A. 0
B. 1
C. 2
D. 3
E. 5
13. Simplify: $\quad \frac{4(3-8+11)}{3(9-12+4)}+2^{3}-8^{2}$
A. -48
B. -2
C. -24
D. -96
E. -56
14. Radley starts jogging at 9:06 am. Radley jogs for 20 minutes, takes a 45 minute break and then jogs for another 1.5 hours. At what time did Radley complete his workout?
A. 11:41 am
B. $11: 37 \mathrm{am}$
C. 11:31 am
D. 11:23 am
E. 11:33 am
15. What is the sum of the prime numbers between 60 and 80 , inclusive?
A. 337
B. 347
C. 341
D. 355
E. 351
16. A number is randomly chosen from 1 to 50 , inclusive. What is the probability the number chosen is a multiple of 3 ?
A. $16 \%$
B. $32 \%$
C. $20 \%$
D. $30 \%$
E. $28 \%$
17. What is the $101^{\text {st }}$ term of the sequence?

99, 92, 85, 78, ...
A. -601
B. -594
C. -587
D. -608
E. -580
18. Two angles are supplementary. The measure of one angle is 40 degrees more than the measure of the other angle. What is the measure of the complement of the smaller angle?
A. $10^{\circ}$
B. $20^{\circ}$
C. $24^{\circ}$
D. $18^{\circ}$
E. $16^{\circ}$
19. Let $\{a, b, c\}$ be the range values of the function $f(x)=3 x-7$, when the domain is $\{-3,0,5\}$. What is the value of $a+b+c$ ?
A. 6
B. -23
C. -15
D. -1
E. -31
20. Mark's bill before taxes was $\$ 84.00$, and after taxes his total bill was $\$ 90.72$. What was the tax rate Mark had to pay?
A. $6.5 \%$
B. $7.5 \%$
C. $7 \%$
D. $8 \%$
E. $8.5 \%$
21. What is the value of $x$ ?

A. 40
B. 60
C. 80
D. 55
E. 85
22. You buy a new jet-ski for $\$ 16,000$, but its value depreciates by $1 / 2$ each year. What will be the value of your jet-ski after 4 years?
A. $\$ 4,000$
B. $\$ 8,000$
C. $\$ 2,000$
D. $\$ 1,000$
E. $\$ 500$
23. $224+225+226+\ldots+240+241+242=$ $\qquad$
A. 4,018
B. 4,427
C. 4,203
D. 4,670
E. 4,671
24. What is the largest unattainable sum of the numbers 8 and 15 ?
A. 91
B. 97
C. 101
D. 111
E. 117
$25.84^{2}=$ $\qquad$ (nearest ten).
A. 7,100
B. 7,050
C. 7,080
D. 7,060
E. 7,000
26. If $m \odot n=\frac{m n}{m+n}$, then what is the value of $(6 \odot 12) \odot(-2 \odot 6)$ ?
A. -12
B. 12
C. 6
D. -6
E. 8
27. Solve for $n$ :
$-8 n+15 \geq-49$
A. $n \geq 4.25$
B. $n \leq-4.25$
C. $n \geq-8$
D. $n \leq-12$
E. $n \leq 8$
28. If $\pi=3.1$, what is the distance around the shape?

A. 55.8 cm
B. 111.6 cm
C. 27.9 cm
D. 37.2 cm
E. 45.9 cm
29. $4,000-1,637=$ $\qquad$ (roman numeral)
A. MMCCCLX
B. MMCCCLXIII
C. MCCCXLIII
D. MMCCCXLVII
E. MMCCCLXVII
30. The graph of the linear equation $y=3 / 4 x-4$ is translated down 11 units. What are the coordinates of the new $y$-intercept of the graph?
A. $(-15,0)$
B. $(7,0)$
C. $(0,15)$
D. $(0,-15)$
E. $(0,-7)$
31. If two cards are randomly drawn without replacement from a standard deck of cards, what is the probability that they will both be 5 's?
A. $\frac{4}{663}$
B. $\frac{1}{1,326}$
C. $\frac{1}{221}$
D. $\frac{1}{13}$
E. $\frac{1}{17}$
32. Simplify: $\quad \frac{\left(5 x^{4} y^{-2}\right)^{-3}}{\left(8 x^{3} y^{2}\right)^{-2}}$
A. $\frac{64 x^{6}}{125 y^{12}}$
B. $\frac{125 x^{6}}{64 y^{12}}$
C. $\frac{64 y^{6}}{125 x^{6}}$
D. $\frac{64 y^{10}}{125 x^{6}}$
E. $\frac{125 y^{10}}{64 x^{6}}$
33. $33_{5} \times 14_{5}=$ $\qquad$ (base 10)
A. 154
B. 164
C. 162
D. 174
E. 462
34. Necklace Necessities Jewelers is designing a new necklace pendant. They start by drawing a circle with a diameter of 30 mm and then cutting out two smaller identical circles each with a diameter of 8 mm . What is the area of the remaining pendant? Let $\pi=3$.

A. $627 \mathrm{~mm}^{2}$
B. $579 \mathrm{~mm}^{2}$
C. $483 \mathrm{~mm}^{2}$
D. $675 \mathrm{~mm}^{2}$
E. $531 \mathrm{~mm}^{2}$
35. Point $W$ has coordinates $(-7,9)$ and is rotated $180^{\circ}$ counterclockwise. What are point $W$ 's new coordinates?
A. $(7,-9)$
B. $(-7,-9)$
C. $(-9,7)$
D. $(-9,-7)$
E. $(7,9)$
36. What is the sum of all integers that satisfy $-5 \leq n<9$ ?
A. 30
B. 45
C. 36
D. 26
E. 21
37. Letter tiles spelling the word KANSAS are placed inside a bag. How many different sequences of letters can be formed using all the letters of KANSAS?
A. 720
B. 180
C. 360
D. 120
E. 540
38. A windshield wiper blade is 12 inches long. In terms of $\pi$, find the area covered by the blade as it rotates through an angle of $120^{\circ}$.
A. $144 \pi$ in $^{2}$
B. $72 \pi \mathrm{in}^{2}$
C. $36 \pi \mathrm{in}^{2}$
D. $48 \pi \mathrm{in}^{2}$
E. $64 \pi \mathrm{in}^{2}$
39. Point $P$ is the midpoint of $\overline{M N}$. Point $M$ has coordinates $(12,8)$ and point $P$ has coordinates $(5,4)$. What is the sum of the coordinates of point $N$ ?
A. -4
B. 0
C. -2
D. 4
E. -6
40. Which of the following equations is the correct equation of a circle with center $(-3,0)$ and has a radius of 22 inches?
A. $(x-3)^{2}+y^{2}=44$
B. $(x+3)^{2}+y^{2}=484$
C. $(x-3)^{2}+y^{2}=88$
D. $(x+3)^{2}+y^{2}=44$
E. $(x-3)^{2}+y^{2}=484$
41. Simplify:
$64^{\frac{2}{3}}+\sqrt{\sqrt{256}}$
A. 20
B. 32
C. 24
D. 16
E. 8
42. A right isosceles triangle has an area of $49 \mathrm{~mm}^{2}$. What is the perimeter of the triangle?
A. 28 mm
B. $14 \sqrt{2} \mathrm{~mm}$
C. $28 \sqrt{2} \mathrm{~mm}$
D. $14+14 \sqrt{2} \mathrm{~mm}$
E. $14+28 \sqrt{2} \mathrm{~mm}$
43. If $a^{3}+b^{3}=(a+b)\left(a^{2}-a b+b^{2}\right)$, then $8 x^{3}+125$ factors to the product of $2 x+5$ and which of the following?
A. $4 x^{2}+10 x-25$
B. $4 x^{2}-10 x-25$
C. $4 x^{2}+10 x+25$
D. $4 x^{2}-10 x+25$
E. $-4 x^{2}+10 x-25$
44. What is the area of the regular hexagon?

A. $150 \sqrt{3} \mathrm{~cm}^{2}$
B. $75 \sqrt{3} \mathrm{~cm}^{2}$
C. $300 \sqrt{3} \mathrm{~cm}^{2}$
D. $225 \sqrt{3} \mathrm{~cm}^{2}$
E. $175 \sqrt{3} \mathrm{~cm}^{2}$
45. The graph of which of the following quadratic equations has its vertex located in the fourth quadrant?
A. $y=x^{2}+8 x+12$
B. $y=x^{2}-6 x+9$
C. $y=2 x^{2}-4 x-3$
D. $y=2 x^{2}+8 x-28$ E. $y=x^{2}-14 x+150$
46. What is the remainder when $m^{2}+11 m+18$ is divided by $m+6$ ?
A. 12
B. 5
C. -1
D. -5
E. -12
47. Which equation can be used to solve for $x$ ?

A. $x=\sin ^{-1}\left(\frac{48}{30}\right)$
B. $x=\cos ^{-1}\left(\frac{48}{30}\right)$
C. $x=\tan ^{-1}\left(\frac{30}{48}\right)$
D. $x=\sin ^{-1}\left(\frac{30}{48}\right)$
E. $x=\cos ^{-1}\left(\frac{30}{48}\right)$
48. If $x+\frac{1}{x}=14$, then what is the value of $x^{2}+\frac{1}{x^{2}}$ ?
A. 196
B. 198
C. 224
D. 146
E. 194
49. How many three-digit positive integers have the property that the middle digit is the mean of the other two digits?
A. 45
B. 55
C. 35
D. 65
E. 50
50. White truffles worth $\$ 40$ per ounce and black truffles worth $\$ 60$ per ounce are to be mixed together to create a mixture of 50 ounces worth $\$ 48$ per ounce. How many ounces of black truffles will be needed?
A. 25 ounces
B. 35 ounces
C. 30 ounces
D. 20 ounces
E. 15 ounces

| 1. B | 18. B | 35. A |
| :---: | :---: | :---: |
| 2. B | 19. C | 36. E |
| 3. C | 20. D | 37. B |
| 4. A | 21. C | 38. D |
| 5. A | 22. D | 39. C |
| 6. C | 23. B | 40. B |
| 7. E | 24. B | 41. A |
| 8. A | 25. D | 42. D |
| 9. A | 26. A | 43. D |
| 10. C | 27. E | 44. A |
| 11. B | 28. E | 45. C |
| 12. C | 29. B | 46. E |
| 13. A | 30. D | 47. D |
| 14. A | 31. C | 48. E |
| 15. E | 32. D | 49. A |
| 16. B | 33. C | 50. D |
| 17. A | 34. B |  |

20. We are trying to find the percent of change. $\frac{90.72}{84}=0.08=8 \%$.
21. First, draw a third line segment parallel to the other two parallel line segments, creating angles $m$ and $n . \angle m$ is
 congruent to the $30^{\circ}$ angle because they are alternate interior angles, so its measure is also $30^{\circ} . \angle n$ is congruent to the $50^{\circ}$ angle because they are alternate interior angles, so its measure is also $50^{\circ}$. Therefore, the value of $x$ equals $30+50=80$.
22. To find the sum of a set of consecutive integers, use the formula $\frac{N(F+L)}{2}$, where $N=$ numbers of terms in the set, $F=$ first term and $L=$ last term. We are asked to find the sum of $224+225+226+\ldots+240+241+242$, so $N=29$, $F=224$ and $L=242$. Thus, $\frac{19(224+242)}{2}=\frac{19(466)}{2}=\frac{8854}{2}=4,427$.
23. The linear equation $y=3 / 4 x-4$ is in slope-intercept form, so the $y$-intercept is -4 . Translating the equation down 11 units means subtracting 11 from -4 , which is -15 . The coordinates of the new $y$-intercept is $(0,-15)$.
24. If a point $(x, y)$ is rotated $180^{\circ}$ counterclockwise, then the new coordinates are $(-x,-y)$. We are given the point $(-7,9)$, so after a rotation of $180^{\circ}$ counterclockwise, then the point's new coordinates are $(7,-9)$.
25. The integers that satisfy $-5 \leq n<9$, are $-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7$ and 8 . Therefore, $-5+(-4)+$ $(-3)+(-2)+(-1)+0+1+2+3+4+5+6+7+8=21$.
26. Since $\frac{120}{360}=\frac{1}{3}$, we are asked to find the area of the region that is $1 / 3$ the area of the entire circle. Our circle has a radius
 of 12 cm . So, the formula we want to use is $A=\frac{\pi r^{2}}{3}$. Substitute our given information and

$$
A=\frac{\pi r^{2}}{3}=\frac{\pi \cdot 12^{2}}{3}=\frac{144 \pi}{3}=48 \pi \mathrm{~cm}^{2} .
$$

41. $64^{\frac{2}{3}}+\sqrt{\sqrt{256}}=(\sqrt[3]{64})^{2}+\sqrt{\sqrt{256}}=4^{2}+\sqrt{16}=16+4=20$.
42. The formula for the area of the regular hexagon is $A=\frac{a P}{2}$, where $a=$ apothem and $P=$ perimeter of the hexagon. Substituting into the formula and $A=\frac{a P}{2}=\frac{5 \sqrt{3}(60)}{2}=\frac{300 \sqrt{3}}{2}=150 \sqrt{3} \mathrm{~cm}^{2}$.
43. We are given $x+\frac{1}{x}=14$. First, square both sides, $\left(x+\frac{1}{x}\right)^{2}=14^{2} .\left(x+\frac{1}{x}\right)^{2}=\left(x+\frac{1}{x}\right)\left(x+\frac{1}{x}\right)=$ $x^{2}+\frac{x}{x}+\frac{x}{x}+\frac{1}{x^{2}}=x^{2}+1+1+\frac{1}{x^{2}}=x^{2}+\frac{1}{x^{2}}+2$. We now see that $x^{2}+\frac{1}{x^{2}}+2=196$. Subtract 2 from both sides and $x^{2}+\frac{1}{x^{2}}=194$.
44. If we have the three-digit positive integer, $a b c$, then we are looking for all three-digit positive integers such that $b=\frac{a+c}{2}$. We know $b$ must be an integer, so we are only looking value of $a$ and $c$ that have an integer value as their mean. $b$ is only an integer when $a+c$ is even. If $a$ is even, $c$ needs to be even also and when $a$ is odd, $c$ must also be odd. If $a$ and $c$ are both even, $a$ can be any of $2,4,6$ and 8 , so four options and $c$ can be any of $0,2,4,6$ and 8 , so five options. This means there are $4 \cdot 5=20$ combinations. If $a$ and $c$ are both odd, they can be any of $1,3,5,7$ and 9 . This gives 5 options for each, so there are $5 \cdot 5=25$ combinations. So, there are $20+25=45$ three-digit positive integers that have their middle digit the mean of the other two digits.
