

# TMSCA MIDDLE SCHOOL MATHEMATICS 

TEST \#13 ©
FEBRUARY23,2019

## GENERAL DIRECTIONS

1. About this test:
A. You will be given 40 minutes to take this test.
B. There are 50 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet be sure to use BLOCK CAPITAL LETTERS. Clean erasures are necessary for accurate grading on Scantrons and Chatsworth cards.
3. If you are using a Chatsworth or Scantron card, please follow the specific instructions given at your particular meet.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have ONE and ONLY ONE correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators MAY NOT be used on this test.
8. All problems answered correctly are worth FIVE points. TWO points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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1. $0.4+0.9+0.8=$ $\qquad$ (nearest tenth)
A. 2.0
B. 2.2
C. 2.1
D. 2.3
E. 2.4
2. $-987-(-692)=$ $\qquad$ C. -285
D. -295
E. -349
A. 1,679
B. $-1,679$
3. $\frac{7}{12} \div 3 \frac{1}{2}=$
A. $1 . \overline{6}$
B. $1.1 \overline{6}$
C. 0.167
D. $0.1 \overline{9}$
E. $0.1 \overline{6}$
4. $78.3 \times 1.11=$ $\qquad$ (nearest integer)
A. 87
B. 80
C. 88
D. 86
E. 85
5. Simplify:
$2 / 3(-18)+1 / 8(-32)-3 / 4\left(-4^{3}\right)+(-7)$
A. -14
B. 24
C. 18
D. -18
E. 25
6. 126 inches $=$ $\qquad$ yards
A. 7.5
B. 3.25
C. 3.5
D. 3.75
E. 10.5
7. How many positive integral divisors less than 50 does the number 280 have?
A. 14
B. 16
C. 10
D. 12
E. 18
8. What is the area of the triangle below?

A. 5.5 units $^{2}$
B. 6.5 units $^{2}$
C. 4.75 units $^{2}$
D. 5.25 units $^{2}$
E. 6 units $^{2}$
9. Store $A$ sells eight towels for $\$ 36.16$ and Store $B$ sells ten towels for $\$ 46.40$. How much more is the unit rate per towel of Store $B$ than Store $A$ ?
A. $\$ 10.24$
B. $8 \not \subset$
C. $16 \phi$
D. $14 \phi$
E. $12 \phi$

10 . What is $15 \%$ of $10 \%$ of 600 ?
A. 150
B. 15
C. 9
D. 12
E. 6
11. If $a \# b=(a+b)^{2}-a b$, what is the value of $-2 \# 6$ ?
A. 28
B. 26
C. 76
D. 52
E. 4
12. Find the number whose cube root is the square of 8 .
A. 512
B. 256
C. 2
D. 16
E. 4
13. Simplify: $\quad 4(n-3)-6(-5-n)$
A. $-2 n-42$
B. $10 n+18$
C. $-2 n-11$
D. $10 n-7$
E. $-2 n-7$
14. What is the sum of all the integers that satisfy the inequality $3 \leq x<9$ ?
A. 42
B. 39
C. 33
D. 49
E. 30
15. How many total degrees are in a regular 17-sided polygon?
A. $2,700^{\circ}$
B. $3,060^{\circ}$
C. $2,880^{\circ}$
D. $2,520^{\circ}$
E. $3,240^{\circ}$
16. $\mathrm{XXXIV}+\mathrm{XLVII}=$ $\qquad$ (Arabic number)
A. 77
B. 83
C. 79
D. 81
E. 73
17. The median of 9 consecutive odd integers is 9 . What is the product of the fourth and eighth numbers?
A. 153
B. 39
C. 119
D. 85
E. 105
18. What is the interquartile range of the set of numbers $75,83,83,83$ and 99 ?
A. 24
B. 12
C. 16
D. 8
E. 15
19. 150 students were asked their favorite ice-cream flavor. The results are shown in the graph below. How many more students said Milk \& Cookies was their favorite flavor than Cookiedough?

A. 12
B. 7
C. 9
D. 6
E. 15
20. If $A=\{1,2,3,4,5,6\}$ and $B=\{3,4,5,6,7,8,9\}$, then $A \cap B$ has how many subsets?
A. 512
B. 511
C. 16
D. 8
E. 32
21. In how many ways can the letters of the word $C H A L K Y$ be arranged?
A. 24
B. 120
C. 21
D. 360
E. 720
22. $\left(3.4 \times 10^{4}\right)\left(4.5 \times 10^{6}\right)=$ $\qquad$ (scientific notation)
A. $1.53 \times 10^{10}$
B. $15.3 \times 10^{10}$
C. $1.53 \times 10^{24}$
D. $1.53 \times 10^{11}$
E. $153 \times 10^{9}$
23. Two sides of a triangle are 7 cm and 15 cm . What is the smallest possible integral length of the third side?
A. 9 cm
B. 8 cm
C. 10 cm
D. 21 cm
E. 13 cm
24. The ratio $a: b=3: 5$ and the ratio $b: c=4: 9$. What is the ratio of $a: c$ ?
A. $1: 3$
B. $4: 15$
C. $4: 45$
D. 7:20
E. 3:20
25. What is the remainder when 98,544 is divided by 14 ?
A. 9
B. 11
C. 10
D. 7
E. 12
26. Which of the following points does not lie on the line $5 x+4 y=-12$ ?
A. $(-12,12)$
B. $(-16,17)$
C. $(-24,-27)$
D. $(-4,2)$
E. $(12,-18)$
27. What is the mean absolute deviation of the numbers $46,49,50,58$ and 62 ?
A. 5.6
B. 4.7
C. 5.9
D. 4.8
E. 5.4
28. What is $3.2 \overline{31}$ expressed as an improper fraction?
A. $\frac{3128}{999}$
B. $\frac{52}{15}$
C. $\frac{319}{90}$
D. $\frac{3199}{990}$
E. $\frac{359}{110}$
29. If $f(x)=3 x-7, g(x)=2-3 x$ and $h(x)=x^{2}$, what is the value of $g(f(h(-3)))$ ?
A. 77
B. -31
C. -58
D. 104
E. -55
30. What is the $6^{\text {th }}$ term of the sequence?
$4, \frac{4}{3}, \frac{4}{9}, \ldots$
A. $\frac{4}{81}$
B. $\frac{4}{243}$
C. $\frac{4}{729}$
D. $\frac{4}{216}$
E. $\frac{4}{2,187}$
31. What is the surface area of a sphere with a radius of 5 inches? Let $\pi=3$.
A. $1,500 \mathrm{in}^{2}$
B. $120 \mathrm{in}^{2}$
C. $500 \mathrm{in}^{2}$
D. $240 \mathrm{in}^{2}$
E. $300 \mathrm{in}^{2}$
32. Milly has a bag that has one red jelly bean, one blue jelly bean and one yellow jelly bean. Nana has a bag with one red jelly bean, one yellow jelly bean and two blue jelly beans. They made an agreement that they would only start eating their jelly beans if they both reached into their bags and pulled out the same color. What is the probability they pull out the same color on their first draw?
A. $1 / 2$
B. $1 / 4$
C. $2 / 3$
D. $1 / 3$
E. $3 / 4$
33. What is the product of the GCF and LCM of the two monomials $36 a^{4} b^{2}$ and $63 a^{3} b^{5}$ ?
A. $2,268 a^{4} b^{5}$
B. $2,268 a^{3} b^{2}$
C. $2,268 a^{7} b^{7}$
D. $2,268 a^{12} b^{10}$
E. $2,268 a b$
34. What is the area of an equilateral triangle with an altitude of 9 inches?
A. $40 \sqrt{5} \mathrm{in}^{2}$
B. $27 \sqrt{3} \mathrm{in}^{2}$
C. $36 \sqrt{2}$ in $^{2}$
D. $36 \sqrt{3} \mathrm{in}^{2}$
E. $18 \sqrt{6}$ in $^{2}$
35. $22_{3}+44_{5}+66_{7}=-10$
A. 99
B. 132
C. 88
D. 110
E. 80
36. What is the sum of the coordinates of the center of the circle with equation $x^{2}+6 x+9=2 y-y^{2}$ ?
A. -4
B. 6
C. -2
D. 5
E. -5
37. What is the area of the trapezoid?

32 inches

A. $576 \mathrm{in}^{2}$
B. $290 \mathrm{in}^{2}$
C. $136 \mathrm{in}^{2}$
D. $440 \mathrm{in}^{2}$
E. $528 \mathrm{in}^{2}$
38. Which of the following is the equation of a line that is perpendicular to the $y$-axis and passes through the point $(5,6)$ ?
A. $y=6$
B. $x=6$
C. $y=5$
D. $y=x+1$
E. $x=5$
39. What is the area of a quadrilateral with vertices located at $(-4,-1),(8,-3),(-2,-6)$ and $(-9,-5)$ ?
A. 42.5 units $^{2}$
B. 21.25 units $^{2}$
C. 44.5 units $^{2}$
D. 54.5 units $^{2}$
E. 85 units $^{2}$
40. David is doing a project for school. He invests $\$ 10$ into an account at $11 \%$ compounded monthly for 2 years. Which function can be used to calculate the amount of money David will have after the 2 years?
A. $y=10\left(1+\frac{0.11}{2}\right)^{24}$
B. $y=10\left(1+\frac{0.11}{12}\right)^{2}$
C. $y=10\left(1+\frac{0.11}{12}\right)^{24}$
D. $y=10\left(\frac{1+0.11}{12}\right)^{24}$
E. $y=10\left(\frac{1+0.11}{12}\right)^{2}$
41. $\log _{6}\left(x^{2} y^{3}\right)$ can be rewritten as which of the following?
A. $\frac{\log _{6} x^{2}}{\log _{6} y^{3}}$
B. $\log _{6}\left(x^{2}+y^{3}\right)$
C. $2 \log _{6} x-3 \log _{6} y$
D. $2 \log _{6} x-\log _{6} y^{3}$
E. $2 \log _{6} x+3 \log _{6} y$
42. In terms of $\pi$ what is the length of a $90^{\circ}$ arc of a circle with a radius of 10 inches?
A. $10 \pi$ inches
B. $25 \pi$ inches
C. $5 \pi$ inches
D. $2.5 \pi$ inches
E. $2.25 \pi$ inches
43. The equation $\sqrt{5 n+2 n-48-6}+6=n$ has solutions $a$ and $b$. Find the value of $2 a b$.
A. 90
B. 180
C. 60
D. 120
E. 160
44. $\triangle A B C$ has vertices with coordinates $(4,0),(17,0)$ and $(6,10) . \triangle X Y Z$ is created by switching the $x$-coordinate and $y$-coordinate of each vertex of $\triangle A B C$. What is the percent of change between the area of $\triangle A B C$ to the area of $\triangle X Y Z$ ?
A. $0 \%$
B. $25 \%$
C. $50 \%$
D. $75 \%$
E. $100 \%$
45. If the pattern continues, how many stars will be needed for Stage $100 ?$


Stage $1 \quad$ Stage 2
Stage 3
Stage 100
A. 389
B. 393
C. 401
D. 385
E. 397
46. If $2^{a}+2^{4}=24,3^{b}+56=65$ and $4^{c}+5^{4}=627$, what is the product of $a \cdot b \cdot c$ ?
A. 12
B. 18
C. 24
D. 3
E. 6
47. What is the sum of the roots of the cubic equation $x^{3}+2 x^{2}-6 x-10=0$ ?
A. $-1 / 2$
B. $-1 / 4$
C. -2
D. -4
E. -14
48. $\left(\frac{14 a^{3} b^{5}}{10 c^{6}}\right)\left(\frac{3 a b c^{4}}{4 a^{3} b}\right)\left(\frac{5 b^{3} c}{7 a^{2} b^{2} c^{3}}\right)=$ $\qquad$
A. $\frac{6 a c^{3}}{7 b^{2}}$
B. $\frac{3 b^{6}}{4 a c^{4}}$
C. $\frac{3 b^{5}}{4 a^{2} c^{3}}$
D. $\frac{3 a c^{3}}{4 b^{2}}$
E. $\frac{6 b^{6}}{7 a c^{4}}$
49. What is the measure of $x$ ?
A. 15
B. 30
C. 8
D. 18
E. 12
50. Thomas is a teacher and bought 18 red and blue pens for the upcoming school year. Thomas paid $\$ 1$ more for each blue pen than he did for each red pen. Without tax, Thomas paid a total of $\$ 64.00$. How many blue pens did Thomas buy?
A. 8
B. 10
C. 6
D. 12
E. 9

| 1. C | $18 . \mathrm{B}$ | $35 . \mathrm{E}$ |
| :--- | :---: | :---: |
| 2. D | $19 . \mathrm{C}$ | $36 . \mathrm{C}$ |
| 3. E | $20 . \mathrm{C}$ | $37 . \mathrm{D}$ |
| 4. A | $21 . \mathrm{E}$ | $38 . \mathrm{A}$ |
| 5. E | $22 . \mathrm{D}$ | $39 . \mathrm{C}$ |
| 6. C | $23 . \mathrm{A}$ | $40 . \mathrm{C}$ |
| 7. | $24 . \mathrm{B}$ | $41 . \mathrm{E}$ |
| 8. A | $25 . \mathrm{E}$ | $42 . \mathrm{C}$ |
| 9. E | $26 . \mathrm{C}$ | $43 . \mathrm{B}$ |
| 10. C | $27 . \mathrm{A}$ | $44 . \mathrm{A}$ |
| 11. A | $28 . \mathrm{D}$ | $45 . \mathrm{E}$ |
| 12. E | $29 . \mathrm{C}$ | $46 . \mathrm{D}$ |
| 13. B | $30 . \mathrm{B}$ | $47 . \mathrm{C}$ |
| 14. C | $31 . \mathrm{E}$ | $48 . \mathrm{B}$ |
| 15. A | $32 . \mathrm{D}$ | $49 . \mathrm{E}$ |
| 16. | $33 . \mathrm{C}$ | $50 . \mathrm{B}$ |
| 17. E | $34 . \mathrm{B}$ |  |



The area of the entire square is 16 units $^{2}$. To find the area of the triangle, subtract the areas of triangles A, B, C and rectangle D. The area of triangle A is $\underline{\underline{1 / 2}(2)(3)=3}$ units $^{2}$. The area of triangle $B$ is $\underline{\underline{1 / 2}}(1)(3)=1.5$ units $^{2}$. The area of triangle $C$ is
 the desired triangle is $16-3-1.5-2-4=5.5$ units $^{2}$.
21. There are 6 different letters in the word $C H A L K Y$, then there are $6!=720$ ways the letters can be arranged.
22. $\left(3.4 \times 10^{4}\right)\left(4.5 \times 10^{6}\right)=15.3 \times 10^{4+6}=15.3 \times 10^{10}=1.53 \times 10^{1} \times 10^{10}=1.53 \times 10^{11}$.
23. If given two sides of a triangle, $a$ and $b$, the smallest possible integral length of the third side can be found by $|a-b|+1$. We are given side lengths of 7 cm and 15 cm . Therefore, the smallest integral length of the third side is $|a-b|+1=|7-15|+1=|-8|+1=8+1=9 \mathrm{~cm}$.
24. $a: b=3: 5$ and $b: c=4: 9$. Find the least common multiple of 4 and 5 , which is 20 . If $b=20$, then $a: b=12: 20$ and $b: c=20: 45$. Therefore, $a: c=12: 45=4: 15$.
32. The probability they each pull out a red jelly bean is $\frac{1}{3} \cdot \frac{1}{4}=\frac{1}{12}$. The probability they each pull out a yellow jelly bean is $\frac{1}{3} \cdot \frac{1}{4}=\frac{1}{12}$. The probability they each pull out a blue jelly bean is $\frac{1}{3} \cdot \frac{1}{2}=\frac{1}{6}$. Therefore, the probability they each pull out the same color of jelly bean is $\frac{1}{12}+\frac{1}{12}+\frac{1}{6}=\frac{4}{12}=\frac{1}{3}$.
47. The sum of the roots of a cubic equation $a x^{3}+b x^{2}+c x+d=0$ can be found by $\frac{-b}{a}$. We are given the equation $x^{3}+2 x^{2}-6 x-10=0$, so $a=1$ and $b=2$. By substituting, the sum of the roots of the equation is $\frac{-b}{a}=\frac{-2}{1}=-2$.
48. $\left(\frac{14 a^{3} b^{5}}{10 c^{6}}\right)\left(\frac{3 a b c^{4}}{4 a^{3} b}\right)\left(\frac{5 b^{3} c}{7 a^{2} b^{2} c^{3}}\right)=\left(\frac{7 a^{3} b^{5}}{5 c^{6}}\right)\left(\frac{3 c^{4}}{4 a^{2}}\right)\left(\frac{5 b}{7 a^{2} c^{2}}\right)=\frac{7 \cdot 3 \cdot 5 \cdot a^{3} \cdot b^{5} \cdot c^{4} \cdot b}{5 \cdot 4 \cdot 7 \cdot c^{6} \cdot a^{2} \cdot a^{2} \cdot c^{2}}=\frac{3 a^{3} b^{6} c^{4}}{4 a^{4} c^{8}}=\frac{3 b^{6}}{4 a c^{4}}$.
49.


The altitude of a right triangle from its right angle to its hypotenuse creates three similar triangles. From the picture, you can create the proportion $\frac{a}{x}=\frac{x}{b}$. Cross multiply and get $x^{2}=a b$ and then square root both sides and $x=\sqrt{a b}$. Therefore, using the information given, $x=\sqrt{24 \cdot 6}=\sqrt{144}=12$.
50. Let $R=$ red pens, $B=$ blue pens and $c=$ cost of a red pen. We can create the equations $R+B=18$ and $R c+B(c+1)=64$. Distribute the $2^{\text {nd }}$ equation to get $R c+B c+B=64$. Subtract $B$ from both sides and we get $R c+B c=64-B$. We can factor the left side and get $c(R+B)=64-B$. We know that $R+B=18$, so we can substitute to get $18 c=64-B$. Therefore, $64-B$ must be divisible by 18 , thus $B=10, c=3$ and $A=8$. Thomas bought a total of 10 blue pens.

