

**TMSCA MIDDLE SCHOOL  
MATHEMATICS  
STATE TEST ©  
APRIL 21, 2018**

**GENERAL DIRECTIONS**

- About this test:
  - You will be given 40 minutes to take this test.
  - There are 50 problems on this test.
- All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading on Scantrons and Chatsworth cards.
- If you are using a Chatsworth or Scantron card, please follow the specific instructions given at your particular meet.
- You may write anywhere on the test itself. You must write only answers on the answer sheet.
- You may use additional scratch paper provided by the contest director.
- All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
- Calculators **MAY NOT** be used on this test.
- All problems answered correctly are worth **FIVE** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
- In case of ties, percent accuracy will be used as a tie breaker.



2017 – 2018 TMSCA Middle School State Mathematics Championship Test

1.  $3\frac{3}{5} + 11\frac{1}{4} + 27\frac{1}{2} =$  \_\_\_\_\_

- A.  $44\frac{3}{20}$       B.  $42\frac{7}{20}$       C.  $43\frac{1}{20}$       D.  $42\frac{1}{20}$       E.  $42\frac{1}{2}$

2.  $7,002 - 4,998 - 47.76 =$  \_\_\_\_\_

- A. 1,956.24      B. -2,772      C. -2,772.24      D. 1,957.24      E. 1,958.24

3.  $4.5 \times 12.8 = 4 \times$  \_\_\_\_\_

- A. 57.6      B. 53.6      C. 18.4      D. 16.2      E. 14.4

4.  $154.8 \div \frac{2}{3} \div 2 =$  \_\_\_\_\_

- A. 132.2      B. 164.4      C. 124.6      D. 116.1      E. 112.3

5. Mike filled up a 2.5 gallon bucket full of water. Mike walked from the water hose to his driveway and spilled 3.5 pints of water. How many ounces were left in Mike's bucket?

- A. 320 ounces      B. 256 ounces      C. 244 ounces      D. 248 ounces      E. 264 ounces

6. 15% of 80% of 2,400 = 40% of what number?

- A. 640      B. 720      C. 760      D. 620      E. 840

7. If you work out the expressions below and place them in increasing numerical order, which expression is the median?

- A.  $4.5 + 6.1$       B.  $17.2 - 9.7$       C.  $8.1 + 2.9$       D.  $14.7 - 6.8$       E.  $11.1 - 0.9$

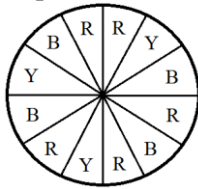
8.  $\frac{5!}{40} =$  \_\_\_\_\_

- A. 3      B. 6      C. 4      D. 5      E. 2

9. What is the next term of the sequence? 0, 2, 4, 6, 12, 22, 40, ...

- A. 74      B. 102      C. 62      D. 84      E. 76

10. Jacky will spin a spinner twice with 5 red sections, 4 blue sections and 3 yellow sections. With all the sections being equal in size, what is the probability that the spinner stops on red first and yellow second?



- A.  $\frac{2}{3}$       B.  $\frac{5}{48}$       C.  $\frac{7}{12}$       D.  $\frac{7}{36}$       E.  $\frac{3}{8}$

11. What is the remainder when 985,424 is divided by the number 13?

- A. 9      B. 10      C. 11      D. 12      E. 0

12. Which of the following is equivalent to the perimeter of a regular hexagon with a side length of 14 feet?

- A. 84 inches      B. 5,004 inches      C. 168 inches      D. 840 inches      E. 1,008 inches

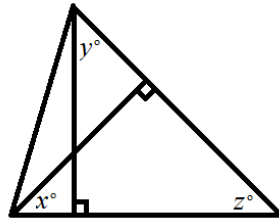
13. Simplify:  $\frac{5-2}{2^3} \div 0.75 + \left(\frac{-2|9-13|}{1-3}\right)^2$

- A.  $16\frac{1}{2}$       B.  $16\frac{1}{4}$       C.  $16\frac{1}{8}$       D.  $16\frac{3}{4}$       E.  $16\frac{3}{8}$

14. What is the unit rate of buying 17 movie tickets for \$161.50?

- A. \$8.75 per ticket      B. \$9.50 per ticket      C. \$8.50 per ticket      D. \$8.25 per ticket      E. \$9.25 per ticket

15. If  $x = 40$ , what is the sum of  $y$  and  $z$ ?



- A. 100                      B. 90                      C. 80                      D. 120                      E. 110

16.  $703 + 704 + 705 + \dots + 729 + 730 + 731 =$  \_\_\_\_\_  
 A. 20,793                      B. 20,197                      C. 20,373                      D. 20,679                      E. 20,693

17. Which of the following is five more than the GCF of the numbers 65 and 78?  
 A. 16                      B. 13                      C. 18                      D. 22                      E. 20

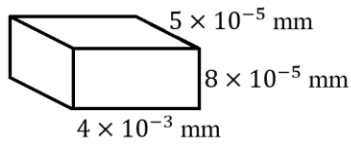
18.  $72^2 - 28^2 =$  \_\_\_\_\_.  
 A. 5,500                      B. 4,400                      C. 5,184                      D. 1,784                      E. 4,936

19. Wendy has six toy ponies, with one being the queen pony. If Wendy wants to arrange them in a line, such that the queen is always first in line, how many different ways can Wendy arrange her ponies?  
 A. 720                      B. 540                      C. 21                      D. 120                      E. 30

20. Tracy's car broke down, so she had to rent a moped to get to work. Tracy rented a moped for \$50 a day plus \$0.40 per mile. Tracy's bill for Monday was \$58.00 and for Tuesday her bill was \$64.00. What is the total number of miles Tracy rode the moped for the two days?  
 A. 65 miles                      B. 122 miles                      C. 58 miles                      D. 60 miles                      E. 55 miles

21. Which polynomials below have a degree of 8?  
 I.  $x^4y^4 + x^3y^3$                       II.  $x^5 + y^3$                       III.  $x + y^8$                       IV.  $x^8y - y$   
 A. I and II                      B. I and III                      C. II and IV                      D. III and IV                      E. I, II, III and IV

22. Find the volume of the rectangular prism.



- A.  $1.048 \times 10^{-6} \text{ mm}^3$     B.  $1.6 \times 10^{-11} \text{ mm}^3$     C.  $1.048 \times 10^{-11} \text{ mm}^3$     D.  $1.6 \times 10^{-6} \text{ mm}^3$     E.  $4.13 \times 10^{-3} \text{ mm}^3$

23. The length of a rectangle is seven more than its width. If the perimeter of the rectangle is 46 inches, what is the measure of the diagonal of the rectangle?  
 A. 15 inches                      B. 16 inches                      C. 17 inches                      D. 18 inches                      E. 20 inches

24. On a TMSCA general mathematics test, Molly attempted 42 problems, getting 4 incorrect. What is Molly's score?  
 A. 176                      B. 194                      C. 184                      D. 182                      E. 188

25. The sum of three consecutive odd integers is 105. What is the value of 10 more than the middle integer?  
 A. 43                      B. 33                      C. 47                      D. 35                      E. 45

26. If  $a \square b = 5ab - 2a - 3b$ , then find the value of  $(1 \square 3) \square (2 \square (1 \square 2))$ ?  
 A. 180                      B. 176                      C. 162                      D. 125                      E. 158

27. An aquarium with dimensions  $6ft \times 5ft \times 8ft$  is only one-fourth full of water. What is the volume of the water?  
 A.  $180\text{ ft}^3$       B.  $120\text{ ft}^3$       C.  $80\text{ ft}^3$       D.  $40\text{ ft}^3$       E.  $60\text{ ft}^3$
28. How many proper subsets can be formed from the set  $\{a, b, c, d, e, f\}$ ?  
 A. 63      B. 64      C. 720      D. 57      E. 39
29. What is the sum of all numbers less than 18 that are relatively prime to 18?  
 A. 48      B. 39      C. 54      D. 46      E. 62
30.  $(1022_3 + 100_6) - (68_9 - 212_3) = \underline{\hspace{2cm}}$  (base 5)  
 A. 44      B. 104      C. 102      D. 112      E. 142
31. Using the magic square below, find the value of  $7(a + b + e)$ .

$2$	$a$	$\frac{2}{3}$
$b$	$1\frac{2}{3}$	$c$
$2\frac{2}{3}$	$d$	$e$

- A.  $14\frac{2}{3}$       B.  $34\frac{1}{3}$       C. 28      D. 42      E. 35
32. Identify which exponential decay function(s) below have a rate of decay greater than 45%.  
 I.  $y = 56(0.82)^x$       II.  $y = 0.24(0.23)^x$       III.  $y = 102\left(\frac{4}{3}\right)^x$       IV.  $y = 763\left(\frac{1}{2}\right)^x$   
 A. I and III      B. II and IV      C. I only      D. I and IV      E. III only
33. How many different sequences of letters can be formed using all the letters of the word ADDITION?  
 A. 10,080      B. 5,040      C. 7,560      D. 11,340      E. 12,600
34. If  $A = x(3x - 2)$ ,  $B = -3x(-2x + 1)$ , and  $C = -2x(5x + 4)$ , find  $A - B + C$ .  
 A.  $-13x^2 - 7x$       B.  $-13x^2 - 13x$       C.  $-13x^2 + 3x$       D.  $-13x^2 - 3x$       E.  $-13x^2 + 7x$
35. Point  $B$  has coordinates  $(-5, 13)$  and is rotated  $270^\circ$  counterclockwise. After  $B$  is rotated, it is then reflected across the  $x$ -axis. What are point  $B$ 's new coordinates?  
 A.  $(-13, 5)$       B.  $(5, 13)$       C.  $(-5, -13)$       D.  $(13, 5)$       E.  $(13, -5)$
36. The angles of a pentagon are in the ratio of 8:8:11:12:15. What is the measure of the median of the angles?  
 A.  $115^\circ$       B.  $110^\circ$       C.  $75^\circ$       D.  $105^\circ$       E.  $125^\circ$
37. The graph of the linear equation  $x - 2y = 10$  is translated eight units up. What is the equation of the new graph?  
 A.  $x - 2y = -6$       B.  $x - 2y = 18$       C.  $x - 2y = 4$       D.  $x - 2y = -4$       E.  $x - 2y = 8$
38. What is the value of the discriminant of the quadratic equation  $0 = (x - 4)^2 + 9$ ?  
 A. -36      B. -84      C. -64      D. -45      E. -27
39. If  $f(x) = x^2 + 3x$  and  $g(x) = -x^2$ , then find  $f(a + b) + g(b - a)$ .  
 A.  $2a^2 + 4ab + 2b^2$       B.  $4ab + 3a + 3b$       C.  $2a^2 + 3a + 3b + 3b^2$       D.  $3a + 3b$       E.  $2a^2 + 4ab + a + b$

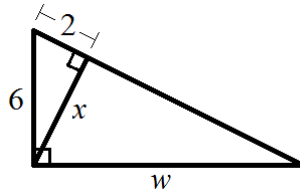
40. Simplify:  $\frac{1}{4} \left( \frac{(6x^3y^{-2})^{-2}(3x^4y^{-5})^2}{(2x^4y^2)^{-3}} \right)^3$

- A.  $8x^{14}$       B.  $\frac{2x^{42}}{y^2}$       C.  $\frac{y^2}{4x^{14}}$       D.  $2x^{42}$       E.  $4x^{42}$

41. Factor:  $8x^3 + 27$

- A.  $(2x - 3)(4x^2 - 6x + 9)$     B.  $(2x - 3)(4x^2 + 6x + 9)$     C.  $(2x + 3)(4x^2 - 6x + 9)$     D.  $(2x + 3)(4x^2 + 6x + 9)$     E. unfactorable

42. Using the picture below, what is the sum of  $w$  and  $x$ ?



- A.  $18\sqrt{2}$       B.  $14\sqrt{2}$       C.  $12\sqrt{2}$       D.  $15\sqrt{2}$       E.  $16\sqrt{2}$

43. The area of a regular hexagon is  $96\sqrt{3}$  in<sup>2</sup> and the perimeter is 48 inches. What is the measure of the apothem?

- A.  $8\sqrt{3}$  inches      B.  $4\sqrt{3}$  inches      C.  $6\sqrt{3}$  inches      D.  $12\sqrt{3}$  inches      E.  $2\sqrt{3}$  inches

44. If  $\sin(x) = \frac{1}{3}$ , then what is the value of  $\cos(x)$ ?

- A.  $\frac{2}{3}$       B.  $\frac{\sqrt{3}}{3}$       C.  $\frac{2\sqrt{2}}{3}$       D.  $\frac{\sqrt{2}}{3}$       E.  $\frac{2\sqrt{3}}{3}$

45. The solution to the system  $\begin{cases} 3x + y = 7 \\ 6x = 12y \end{cases}$  is  $(a, b)$  and the solution to the system  $\begin{cases} y = 8x + 5 \\ 6x - y = -6 \end{cases}$  is  $(c, d)$ . In simplest radical form, what is the value of  $(abd)^c$ ?

- A.  $2\sqrt{3}$       B.  $6\sqrt{2}$       C.  $4\sqrt{3}$       D.  $3\sqrt{6}$       E.  $3\sqrt{2}$

46. If  $x + y = 6$  and  $xy = 2$ , then what is the value of  $x^3 + y^3$ ?

- A. 216      B. 221      C. 191      D. 210      E. 180

47. Simplify:  $\left(1 + \frac{4}{a}\right) \div \left(1 - \frac{16}{a^2}\right)$

- A.  $\frac{1}{a-4}$       B.  $\frac{a}{a^2-2}$       C.  $\frac{1}{a-2}$       D.  $\frac{a}{a-4}$       E.  $\frac{a}{a+2}$

48. What is the sum of the integral solutions of the inequality  $|n + 3| < 9$ ?

- A. -78      B. -51      C. -57      D. -63      E. -68

49. A ball is thrown up into the air from a height of 4 feet. The ball will be  $h$  feet above the ground at  $t$  seconds, and is modeled by the function  $h = -16t^2 + 64t + 4$ . What is the maximum height the ball will reach before it starts descending?

- A. 54 ft      B. 64 ft      C. 68 ft      D. 72 ft      E. 74 ft

50. Which value of  $x$  makes the expression  $\frac{3x-8}{x-5}$  undefined?

- A. 5      B. 8      C.  $\frac{8}{3}$       D. -5      E.  $-\frac{8}{3}$

2017 – 2018 TMSCA Middle School State Mathematics Championship Test Answer Key

1. B	18. B	35. E
2. A	19. D	36. B
3. E	20. E	37. A
4. D	21. B	38. A
5. E	22. B	39. B
6. B	23. C	40. D
7. E	24. D	41. C
8. A	25. E	42. E
9. A	26. C	43. B
10. B	27. E	44. C
11. C	28. A	45. E
12. E	29. C	46. E
13. A	30. D	47. D
14. B	31. C	48. B
15. B	32. B	49. C
16. A	33. A	50. A
17. C	34. A	

18.  $72^2 - 28^2 = (72 + 28)(72 - 28) = (100)(44) = 4,400$ .

24. On a TMSCA general math test, correct problems are worth 5 points and incorrect answers are worth -2 points. Therefore,  $42 - 4 = 38$  correct answers, so  $38(5) = 190$ .  $190 - 2(4) = 182$ . Molly's score will be 182.

25. Let  $x =$  odd integer. We have  $x + x + 2 + x + 4 = 105$ , so  $3x + 6 = 105$  and solving gives  $x = 33$ . Our three consecutive odd integers are 33, 35 and 37. 10 more than the middle integer is  $35 + 10 = 45$ .

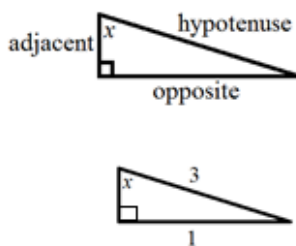
27.  $6ft \times 5ft \times 8ft = 240ft^3$ .  $\frac{1}{4}$  of  $240 = \frac{1}{4}(240) = 60 ft^3$ .

28. To find the number of proper subsets of a set, use  $2^n - 1$ , where  $n$  is equal to the number of elements in the set. We are given the set  $\{a, b, c, d, e, f\}$ , which has 6 elements. Therefore, the number of proper subsets of the given set, is  $2^6 - 1 = 64 - 1 = 63$ . There are 63 proper subsets of the set  $\{a, b, c, d, e, f\}$ .

33. We are trying to find the number of permutations we can create using the letters of the word ADDITION. However, in this word we have repeating letters, D and I. To find the number of different permutations of  $n$  objects where one object repeats  $a$  times and a second object repeats  $b$  times, use  $\frac{n!}{a!b!}$ . So, since there are 8 letters with D repeating twice and I repeating twice, there are  $\frac{8!}{2!2!} = 10,080$  sequences of letters formed using all the letters of the word ADDITION.

36. A pentagon has  $540^\circ$ . Let  $x$  equal the constant multiplier. If the ratio of the pentagon's angles is 8:8:11:12:15, then we have  $8x + 8x + 11x + 12x + 15x = 540 \rightarrow 54x = 540$ . Divide both sides by 54 and  $x = 10$ , which give us the angles  $8(10) = 80$ ,  $8(10) = 80$ ,  $11(10) = 110$ ,  $12(10) = 120$  and  $15(10) = 150$ , so our angles are  $80^\circ, 80^\circ, 110^\circ, 120^\circ$  and  $150^\circ$ . The median of these angles is then  $110^\circ$ .

44. From reference  $\angle x$ , we label our right triangle, as follows.



The trig function sine is equal to the opposite side divided by the hypotenuse. We are given  $\sin(x) = \frac{1}{3}$ . So we have the following triangle (not drawn to scale). Using the Pythagorean Theorem, we find the missing side length to be  $2\sqrt{2}$ . The trig function cosine is equal to adjacent divided by the hypotenuse. Therefore,  $\cos(x) = \frac{2\sqrt{2}}{3}$ .

46. First, cube both sides. If  $x + y = 6$ , then  $(x + y)^3 = 6^3$ .  $6^3 = 216$  and  $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$ . Rearrange  $x^3 + 3x^2y + 3xy^2 + y^3$  to  $x^3 + y^3 + 3x^2y + 3xy^2$ . Factor  $3x^2y + 3xy^2$  to  $3xy(x + y)$ . Since we know that  $xy = 2$  and  $x + y = 6$ , then  $3xy(x + y) = 3(2)(6) = 36$ . We now have  $x^3 + y^3 + 36 = 216$ . Subtract 36 from both sides and  $x^3 + y^3 = 180$ .

47. To simplify  $\left(1 + \frac{4}{a}\right) \div \left(1 - \frac{16}{a^2}\right)$ , simplify  $1 + \frac{4}{a}$  to  $\frac{a}{a} + \frac{4}{a} = \frac{a+4}{a}$  and  $1 - \frac{16}{a^2}$  to  $\frac{a^2}{a^2} - \frac{16}{a^2} = \frac{a^2-16}{a^2}$ . Now, we have  $\frac{a+4}{a} \div \frac{a^2-16}{a^2}$ . When dividing fractions, multiply by the reciprocal of the 2<sup>nd</sup> fraction,  $\frac{a+4}{a} \div \frac{a^2-16}{a^2} = \frac{a+4}{a} \cdot \frac{a^2}{a^2-16}$ . We see that  $a^2 - 16$  is a difference of squares that factors to  $a^2 - 16 = (a + 4)(a - 4)$ . We now have  $\frac{a+4}{a} \cdot \frac{a^2}{(a+4)(a-4)}$  which gives us  $\frac{a^2(a+4)}{a(a+4)(a-4)}$ . Simplify  $\frac{a^2(a+4)}{a(a+4)(a-4)}$ , and we get  $\frac{a^{\cancel{2}}(\cancel{a+4})}{\cancel{a}(\cancel{a+4})(a-4)} = \frac{a}{a-4}$ .

50. The value of  $x$  that makes the expression  $\frac{3x-8}{x-5}$  undefined is 5, because if  $x = 5$ , then the denominator would be equal to 0, and having a denominator of 0 is not allowed.