TMSCA MIDDLE SCHOOL MATHEMATICS TEST #5 ©

NOVEMBER 17, 2018

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 50 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading on Scantrons and Chatsworth cards.
3. If you are using a Chatsworth or Scantron card, please follow the specific instructions given at your particular meet.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators **MAY NOT** be used on this test.
8. All problems answered correctly are worth **FIVE** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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1. $117 + (-67) =$ _____

A. 184 B. -184 C. 50 D. 40 E. -50

2. 100,987 – 48,193 = _____ (nearest hundred)

A. 52,800 B. 52,700 C. 52,780 D. 52,790 E. 52,000

3. $16\frac{3}{8} \times 0.6 =$ _____

A. $9\frac{33}{40}$ B. $9\frac{17}{40}$ C. $8\frac{37}{40}$ D. $8\frac{17}{40}$ E. $9\frac{7}{8}$

4. $6{,}148 \div \frac{1}{2} =$ _____

A. 12,896 B. 12,396 C. 12,286 D. 12,296 E. 3,074

5. What is the prime factorization of the number 1,240?

A. $2^3 \cdot 5^2 \cdot 31$ B. $2^2 \cdot 5 \cdot 13^2$ C. $2^2 \cdot 5 \cdot 31$ D. $2^2 \cdot 5^2 \cdot 31^2$ E. $2^3 \cdot 5 \cdot 31$

6. Simplify: $(8^2 - 4^2) \div 2(18 - 2^3)$

A. 160 B. -16 C. -32 D. 288 E. 240

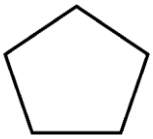
7. Charles can cast his fishing lure eighty-one yards. How many feet can Charles cast his fishing lure?

A. 27 ft B. 162 ft C. 108 ft D. 243 ft E. 202.5 ft

8. Farhan is thinking of two prime numbers. If Farhan's numbers are the two greatest prime numbers less than 70, what is the sum of the two prime numbers Farhan is thinking of?

A. 138 B. 128 C. 144 D. 134 E. 123

9. How many total diagonals can be drawn in the shape below?



A. 10 B. 9 C. 5 D. 2 E. 3

10. Alise has nine quarters, six dimes, eleven nickels and seventeen pennies. Meg has ten quarters, seven dimes, four nickels and five pennies. How much more money does Alise have than Meg?

A. \$1.18 B. \$0.52 C. \$0.12 D. \$0.24 E. \$1.02

11. When all the numbers from zero to forty-six are written, how many of the digits are 2's?

A. 15 B. 14 C. 10 D. 12 E. 13

12. What is the sum of the range and median of the set of numbers {112, 144, 143, 127, 139}?

A. 272 B. 251 C. 175 D. 144 E. 171

13. Point *A* has coordinates $(-27, 19)$ and is translated to point *B* by the rule $(x, y) \rightarrow (x + 33, y - 24)$. What is the sum of the coordinates of point *B*?

A. $(6, -5)$ B. 2 C. $(6, -43)$ D. -37 E. 1

14. An adult ticket to enter *Bouncy World* costs \$6.00. A child ticket costs 20% less than an adult ticket. If a mother and two children enter *Bouncy World*, what was their total entrance fee?

A. \$16.20 B. \$15.40 C. \$14.40 D. \$15.60 E. \$20.40

15. If $A = 1$, $B = 2$, $C = 3$, …, $Y = 25$ and $Z = 26$, what is the sum of the values of the letters of the word *PURPLE*?
A. 88 B. 79 C 81 D. 86 E. 84

16. 0.4375 = _____ (fraction)
A. $\frac{5}{16}$ B. $\frac{3}{8}$ C. $\frac{437}{1000}$ D. $\frac{5}{11}$ E. $\frac{7}{16}$

17. 4! + 19 = _____
A. 43 B. 23 C. 44 D. 35 E. 529

18. Clinton's bowling ball weighs 72 ounces. How many pounds does Clinton's bowling ball weigh?
A. 6 pounds B. 4.5 pounds C. 5.25 pounds D. 6.25 pounds E. 4.75 pounds

19. What is the sum of all the even positive integral divisors of the number 28?
A. 42 B. 56 C. 48 D. 28 E. 46

20. What is the positive difference of the total number of diagonals of a regular hexagon and the number of diagonals that can be drawn from one vertex of a regular pentagon?
A. 2 B. 4 C. 5 D. 7 E. 9

21. Bailey is trying to throw tennis balls into a bucket. He had 9 successful throws and 16 unsuccessful throws. What is the probability Bailey's next throw will be successful?
A. 56.25% B. 36% C. 64.75% D. 42% E. 42.25%

22. What is the complement to an angle that has a measure of 36.4°?
A. 8.6° B. 54.6° C. 143.6° D. 53.6° E. 144.6°

23. If Bertrice continues writing down the word *VALUE* repeatedly, what is the 47th letter Bertrice will write down?
A. *V* B. *A* C. *L* D. *U* E. *E*

24. What is the sum of the digits of the largest palindrome less than 4,046?
A. 16 B. 10 C. 8 D. 14 E. 12

25. What number must be added to the list of numbers 79, 68, 61, 75 and 67 in order for the list to have a median of 71?
A. 72 B. 76 C. 70 D. 74 E. 81

26. A line segment has endpoints with coordinates $(17, -34)$ and $(-35, -28)$. What is the sum of the coordinates of its midpoint?
A. -40 B. -5 C. -18 D. -24 E. 22

27. Sara paid \$13.75 less for a t-shirt than she paid for a pair of pants. Altogether she paid \$44.75 for both (assuming no taxes). What was the cost of the t-shirt Sara bought?
A. \$14.75 B. \$16.25 C. \$17.25 D. \$17.50 E. \$15.50

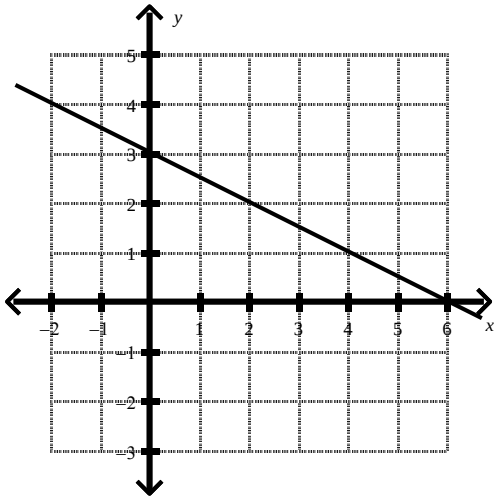
28. If $3x + 17 = 53$, what is the value of $5x - 7$?
A. 53 B. 12 C. 65 D. 47 E. 19

29. What is the area of a rhombus with diagonals measuring 18 cm and 6 cm?
A. 48 cm^2 B. 36 cm^2 C. 60 cm^2 D. 54 cm^2 E. 42 cm^2

30. What is the value of *x*, if $\frac{1}{x}+\frac{1}{x}=6$?

A. ½ B. ⅓ C. -3 D. 3 E. 2

31. What is an equation of the line graphed below?



A. $x-2y=3$ B. $x+2y=6$ C. $2x-y=3$ D. $2x+y=6$ E. $x+y=3$

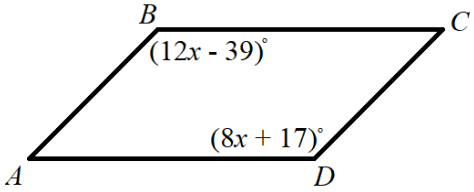
32. How many combinations can be made of 10 items taken 8 at a time?

A. 1,814,400 B. 907,200 C. 45 D. 63 E. 80

33. What is the positive geometric mean of the numbers 9 and 25?

A. 17 B. 15 C. 18 D. 16 E. 16.5

34. *ABCD* is a parallelogram. Find $m\angle C$.



A. 14° B. 129° C. 64.5° D. 51° E. 65°

35. Simplify: $7(5\sqrt{72})$

A. $210\sqrt{2}$ B. $35\sqrt{72}$ C. $70\sqrt{18}$ D. $140\sqrt{6}$ E. $72\sqrt{6}$

36. Which of the following is a linear factor of the polynomial $x^2+2x-48$?

A. *x* – 8 B. *x* + 6 C. *x* – 16 D. *x* + 3 E. *x* + 8

37. What is the growth factor of the exponential growth function $y=99(3.6)^x$?

A. 260 B. 2.6 C. 3.6 D. 99 E. 356.4

38. Point *M* has coordinates (5, 9). If point *M* is rotated 180° about the origin and then translated down seven units and to the left five units. What are the new coordinates of point *M*?

A. $(-10,-16)$ B. $(-12,-14)$ C. $(2,-2)$ D. $(-10,-12)$ E. $(-4,2)$

39. Simplify: $5\sqrt{11^2}$

A. 55 B. 605 C. 24.2 D. $5\sqrt{22}$ E. 110

40. Which of the following is the quadratic equation $y = 2(x-3)^2 + 6$ expressed in standard form?

A. $y = 2x^2 - 12x + 24$ B. $y = 2x^2 - 12x + 18$ C. $y = 2x^2 - 6x + 15$ D. $y = 2x^2 - 12x + 12$ E. $y = 2x^2 - 6x + 12$

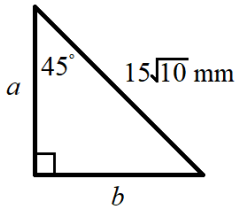
41. If $m(x) = x^3 - 5$ and $n(x) = 2x^2 + 3$, what is the value of $m(3) + n(-3)$?

A. 43 B. 61 C. 1 D. -11 E. 7

42. What is the sum of the coordinates of the solution to the system of linear equations $\begin{cases} y = 2x + 5 \\ x = 2y - 31 \end{cases}$?

A. 12 B. 32 C. 26 D. 29 E. 31

43. Using the picture below, what is the measure of $a + b$?



A. $60\sqrt{5}$ mm B. $15\sqrt{5}$ mm C. $30\sqrt{5}$ mm D. $30\sqrt{3}$ mm E. 45 mm

44. $\log_{10} 8 + \log_{10} 9 =$ _____

A. $\log_{10} 72$ B. $\log_{10} 17$ C. $\log_{10}\left(\frac{8}{9}\right)$ D. $\log_{17} 10$ E. $\frac{\log_{10} 8}{\log_{10} 9}$

45. What is an equation of the circle with its center with coordinates $(-11, 0)$ and a diameter of 22 units?

A. $(x+11)^2 + y^2 = 121$ B. $(x-11)^2 + y^2 = 484$ C. $(x+11)^2 + y^2 = 88$ D. $(x-11)^2 + y^2 = 88$ E. $(x+11)^2 + y^2 = 484$

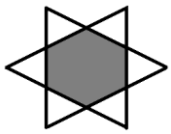
46. What is the sum of the roots of the quadratic equation $y = 3x^2 - 18x + 36$?

A. 12 B. -2 C. 2 D. 6 E. –½

47. Rationalize the denominator: $\frac{5}{2-\sqrt{3}}$ = _____

A. $\frac{10+5\sqrt{3}}{2+\sqrt{3}}$ B. $10 + 5\sqrt{3}$ C. $10 - 5\sqrt{3}$ D. $\frac{10-5\sqrt{3}}{2+\sqrt{3}}$ E. $\frac{10-5\sqrt{3}}{2-\sqrt{3}}$

48. Two congruent equilateral triangles are placed on top of each other so that they form a regular hexagon. If each triangle has an area of 72 cm^2, what is the area of the hexagon?



A. 48 cm^2 B. 36 cm^2 C. 54 cm^2 D. 56 cm^2 E. 52 cm^2

49. Solve for g: $\sqrt{g+2} + 8 = 5$

A. 7 B. 11 C. 14 D. -5 E. no solution

50. What is the sum of the reciprocals of all the factors of 18?

A. $1\frac{5}{6}$ B. 39 C. $2\frac{1}{6}$ D. $2\frac{5}{6}$ E. $2\frac{7}{18}$

2018 – 2019 TMSCA Middle School Mathematics Test #5 Answer Key

1. C
2. A
3. A
4. D
5. E
6. E
7. D
8. B
9. C
10. C
11. A
12. E
13. E
14. D
15. A
16. E
17. A
18. B
19. C
20. D
21. B
22. D
23. B
24. C
25. D
26. A
27. E
28. A
29. D
30. B
31. B
32. C
33. B
34. D
35. A
36. E
37. C
38. A
39. A
40. A
41. A
42. C
43. C
44. A
45. A
46. D
47. B
48. A
49. E
50. C

6. $(8^2 - 4^2) \div 2(18 - 2^3) = (64 - 16) \div 2(18 - 8) = 48 \div 2(10) = 24(10) = 240.$

19. The positive integral divisors of the number 28 are 1, 2, 4, 7, 14 and 28. The even positive integral divisors are then 2, 4, 14 and 28, so 2 + 4 + 14 + 28 = 48.

24. The largest palindrome less than 4,046 is 4,004. Therefore, 4 + 0 + 0 + 4 = 8.

27. Let *x* be equal to the cost of the t-shirt. We can create the equation *x* + *x* + 13.75 = 44.75, and combine like terms to get 2*x* + 13.75 = 44.75. Subtract 13.75 from both sides and 2*x* = 31. Divide both sides by 2 and then *x* = 15.50. The cost of the t-shirt is $15.50.

29. To find the area of a rhombus, use the formula $\frac{d_1 \cdot d_2}{2}$, where d_1 and d_2 are the diagonals of the rhombus. We are given diagonals measuring 18 cm and 6 cm, so substituting into the formula and we get an area of $\frac{18 \cdot 6}{2} = \frac{108}{2} = 54$ cm^2.

30. One way to solve the equation $\frac{1}{x} + \frac{1}{x} = 6$ is to first multiple the entire equation by *x*, and we get $x\left(\frac{1}{x} + \frac{1}{x} = 6\right); so\ \frac{x}{x} + \frac{x}{x} = 6x \to 1 + 1 = 6x \to 2 = 6x.$ Now, divide both sides by 2 and we get *x* = ⅓.

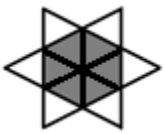
33. The geometric mean of two numbers is equal to the square root of their product, given two numbers *a* and *b*, their geometric mean is $\sqrt{ab}$. Therefore, the geometric mean of 9 and 25 is $\sqrt{9 \cdot 25} = \sqrt{225} = 15.$

40. Standard form of a quadratic equation is $y = ax^2 + bx + c$. To change $y = 2(x - 3)^2 + 6$ into standard form, first Square the $x - 3$, $(x - 3)^2 = x^2 - 3x - 3x + 9 = x^2 - 6x + 9$. Now we have $y = 2(x^2 - 6x + 9) + 6$. Next, distribute the 2, $2(x^2 - 6x + 9) = 2x^2 - 12x + 18$. Finally, combine like terms and $y = 2x^2 - 12x + 18 + 6 = y = 2x^2 - 12x + 24.$

46. A quadratic equation in standard form is $y = Ax^2 + Bx + C$. The sum of the roots can be found by $\frac{-B}{A}$. We are given the quadratic equation $y = 3x^2 - 18x + 36$, so *A* = 3 and *B* = −18. Substituting, and we get the sum of the roots to be $\frac{-(-18)}{3} = \frac{18}{3} = 6.$

47. We must rationalize the denominator by multiplying the denominator by its conjugate, which is $2 + \sqrt{3}$. So, $\frac{5}{2-\sqrt{3}} \cdot \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{5(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} = \frac{10+5\sqrt{3}}{4+2\sqrt{3}-2\sqrt{3}-3} = \frac{10+5\sqrt{3}}{4-3} = \frac{10+5\sqrt{3}}{1} = 10 + 5\sqrt{3}.$

48. To find the area of the regular hexagon, draw lines as such,



We can now see that every small triangle is congruent and equilateral. Therefore, the shaded hexagon is $\frac{6}{9}$ or $\frac{2}{3}$ the area of one of the large equilateral triangles. Since one large triangle has an area of 72 cm^2, the area of the hexagon is then $\frac{2}{3} \cdot 72 = 48$ cm^2