

# TMSCA MIDDLE SCHOOL MATHEMATICS <br> TEST \# 6 © <br> DECEMBER3,2016 

## GENERAL DIRECTIONS

1. About this test:
A. You will be given 40 minutes to take this test.
B. There are 50 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet be sure to use BLOCK CAPITAL LETTERS. Clean erasures are necessary for accurate grading on Scantrons and Chatsworth cards.
3. If you are using a Chatsworth or Scantron card, please follow the specific instructions given at your particular meet.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have ONE and ONLY ONE correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators MAY NOT be used on this test.
8. All problems answered correctly are worth FIVE points. TWO points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

1. Which expression below produces the largest perfect square?
A. $84+172$
B. $148+176$
C. $287+2$
D. $11+351$
E. $144+324$
2. Clint buys a video game for $\$ 34.78$, a remote controller for $\$ 28.99$ and a battery charger for $\$ 8.24$. Without tax, how much change will Clint get back if he pays with $\$ 100.01$ ?
A. $\$ 48.99$
B. \$27.99
C. \$32.01
D. $\$ 28.00$
E. \$32.00
3. $24 \div \frac{2}{3}=$ $\qquad$ (nearest ten)
A. 70
B. 30
C. 36
D. 28
E. 40
4. Sophia and her friends Lilly, Megan, Shayna and Marcy, go to Playland USA. Sophia won the jackpot on one of the games and won 1,200 tickets. She wants to give each of her friends one-fifth of her tickets. Lilly wants to split her tickets amongst her three little sisters by giving them each one-third of her tickets. How many tickets will each of Lilly's sisters receive?
A. 120
B. 60
C. 240
D. 80
E. 100
5. Lewis calculates the product of 321 and 145 to be 46,575 . How much over was his calculation than the actual product of 321 and 145 ?
A. 25
B. 30
C. 35
D. 40
E. 45
6. Mr. T's class of 35 students is getting free lunches as a reward. Each lunch has a sandwich, a bag of chips, a drink and either an apple or orange. If $40 \%$ of the lunches contain an apple, how many lunches contain an orange?
A. 14
B. 17
C. 18
D. 21
E. 24
7. What is the reciprocal of $8 \frac{17}{50}$ written as a decimal and rounded to the nearest hundredths?
A. 0.21
B. 0.18
C. 0.34
D. 0.12
E. 0.14
8. Calculate the volume of the cylinder below, in terms of $\pi$.

A. $1,209.6 \pi \mathrm{~cm}^{2}$
B. $2,640.6 \pi \mathrm{~cm}^{2}$
C. $3,120.6 \pi \mathrm{~cm}^{2}$
D. $586.8 \pi \mathrm{~cm}^{2}$
E. $1,173.6 \pi \mathrm{~cm}^{2}$
9. What is twice the sum of the distinct prime factors of the number 560 ?
A. 20
B. 40
C. 28
D. 14
E. 22
10. Sherri has a bag of 14 red marbles, 8 green marbles, 9 blue marbles and 3 yellow marbles. What is the probability Sherri chooses two marbles, with replacement, and gets a red or green each time?
A. $\frac{11}{17}$
B. $\frac{7}{11}$
C. $\frac{7}{17}$
D. $\frac{121}{289}$
E. $\frac{15}{17}$
11. Kai is given the four numbers, $171,619,118$ and 456 . Using only two numbers at a time to find a sum, what is the largest difference between the largest sum and smallest sum Kai can make of the four numbers?
A. 1,075
B. 173
C. 786
D. 574
E. 737
12. What value is $40 \%$ of $20 \%$ of $1 / 2$ of 400 ?
A. 24
B. 16
C. 40
D. 32
E. 12
13. $\overrightarrow{B D}$ bisects $\angle A B C$ and $m \angle D B C=23.7^{\circ}$. What is the measure of the complement of $\angle A B C$ ?
A. $42.6^{\circ}$
B. $47.4^{\circ}$
C. $47.6^{\circ}$
D. $78.15^{\circ}$
E. $21.3^{\circ}$
14. If $a \pitchfork b=3 a^{2}+2 b^{2}+1$, then find the value of $(4 \pitchfork(-2))+(5 \pitchfork 0)$.
A. 157
B. 176
C. 133
D. 127
E. 163
15. $\overline{K L}$ has endpoints $K(27,11)$ and $L(23,49)$. If point $L$ is reflected across the $y$-axis to create point $M$, what is the sum of the coordinates of the midpoint of $\overline{K M}$ ?
A. 32
B. 55
C. 17
D. 23
E. 28
16. The area of two identical circles combined is $512 \pi$ units $^{2}$. If a square has a side length equal to that of the diameter of one of the circles, what is the perimeter of the square?
A. 64 units
B. 32 units
C. 128 units
D. 256 units
E. 144 units
17. On a number line, $X$ and $Y$ are located at 16 and 40, respectively. $Z$ is the midpoint of $\overline{X Y}$ and $W$ is the midpoint of $\overline{X Z}$. What is the coordinate of $Z$ ?
A. 28
B. 12
C. 24
D. 26
E. 22
18. $-8+(-6)+(-4)+(-2)+\cdots+24=$ $\qquad$
A. 124
B. 148
C. 136
D. 130
E. 154
19. Using the examples below, find the value of $n-13$ ?

A. 22
B. 23
C. 24.5
D. 19.5
E. 25
20. What is the sum of the five greatest prime numbers less than 100 ?
A. 419
B. 395
C. 421
D. 449
E. 473
21. $(24-16)(24+16)=$ $\qquad$ (Roman numeral)
A. MMMVV
B. MMMVIII
C. CCCXX
D. CCCVIII
E. CCCII
22. If $n=2^{3} \cdot 683$, then find the value of the sum of the digits of $n$.
A. 691
B. 25
C. 16
D. 19
E. 21
23. A normal animal cell has a diameter of 0.000000024 mm . If a bacteria cell has a radius half the length of a normal animal cell, what is the bacteria cell's length in scientific notation?
A. $1.2 \times 10^{-8}$
B. $2.4 \times 10^{-8}$
C. $6 \times 10^{-9}$
D. $3 \times 10^{-9}$
E. $1.5 \times 10^{-9}$
24. In terms or $\pi$, what is the perimeter of the shaded region using the picture below?

A. $12 \pi \mathrm{~cm}$
B. $12 \pi+8 \mathrm{~cm}$
C. $12 \pi+16 \mathrm{~cm}$
D. $48 \pi+8 \mathrm{~cm}$
E. $48 \pi+16 \mathrm{~cm}$
25. A square has an area of 576 units $^{2}$. If the square has the exact same perimeter as an equilateral triangle, what is the side length of the equilateral triangle?
A. 16 units
B. 24 units
C. 36 units
D 32 units
E. 28 units
26. $A=4 x-5$ and $B=7 x-6$. Find $6 A-3 B$.
A. $3 x-12$
B. $3 x-48$
C. $3 x+18$
D. $3 x-18$
E. $3 x+13$
27. Using the picture below find the value of $P+Q$.

A. 42 units
B. 30 units
C. 54 units
D. 48 units
E. 38 units
28. To keep up with newer ideas, the CEO of Cell Phone Innovations can hold his position for only five years. What is the minimum number of CEO's Cell Phone Innovations could have during a twenty-year period?
A. 1
B. 2
C. 3
D. 4
E. 5
29. What is the maximum number of intersections points created using only three lines?
A. 0
B. 1
C. 2
D. 3
E. 4
30. If sales tax on a $\$ 16.00 \mathrm{t}$-shirt is $\$ 1.32$, how much would the sales tax be on a $\$ 200.00$ item?
A. $\$ 10.50$
B. $\$ 14.50$
C. $\$ 18.50$
D. $\$ 16.50$
E. $\$ 12.50$
$31.45 \%$ of 24,500 is equal to $20 \%$ of which number?
A. 11,025
B. 21,025
C. 55,125
D. 65,225
E. 42,025
31. Point $A$ has coordinates $(-13,19)$ and is reflected over the $x$-axis and then translated nine units up and eighteen units to the left. What is the product of the new coordinates $A$ ?
A. 247
B. 310
C. -868
D. 50
E. 148
32. Using the picture below, $\overrightarrow{B D}$ bisects $\angle A B C$. Find the measure of $\angle A B C$.

A. $15^{\circ}$
B. $5^{\circ}$
C. $25^{\circ}$
D. $45^{\circ}$
E. $30^{\circ}$
33. What is the measure of the smallest angle in a triangle if one angle measure $100^{\circ}$ and the other two angles of the triangle are in a ratio of 7:9?
A. $55^{\circ}$
B. $35^{\circ}$
C. $40^{\circ}$
D. $30^{\circ}$
E. $25^{\circ}$
34. $22_{7}+44_{7}=$ $\qquad$
A. 66
B. 106
C. 112
D. 111
E. 61
35. Find the value of $\frac{a+b+1}{c}$, if $a, b$ and $c$ are three distinct positive whole numbers, where $c<b<a$ and $a^{2}+b^{2}+c^{2}=121$.
A. 8
B. 2
C. 10
D. 4
E. 6
36. If $f(x)=x^{2}+8$ and $g(x)=14 x-23$, then find the value of $5 f(-3)-2 g(-2)$.
A. 22
B. 187
C. 141
D. 119
E. 136
37. Chords $A B$ and $C D$ intersect in a circle at point $P$. If $\overline{B P}$ has a measure of 3 units, $\overline{C P}$ has a measure of 4 units and $\overline{C D}$ has a measure of 13 units, what is the measure of $\overline{A P}$ ?
A. 10 units
B. $17^{1 / 3}$ units
C. 15 units
D. $13^{2 / 3}$ units
E. 12 units
38. What is the positive difference between the geometric mean and arithmetic mean of the numbers 48 and 12 ?
A. 2
B. 10
C. 14
D. 12
E. 6
39. Marco has a piece of wood measuring 70 inches long. He needs to cut the board into three pieces because he is making a workshop bench for his garage. The longest piece must be twice the length of the middle-sized piece, and the shortest piece must be 10 inches shorter than the middle sized piece. How long must the longest piece of wood be?
A. $3 \frac{1}{3}$ feet
B. 40 feet
C. 20 feet
D. $4 \frac{1}{2}$ feet
E. $\frac{5}{6}$ feet
40. Find the value of $7^{x}+8^{y}$ when $7^{x+2}=294$ and $8^{y+2}=448$.
A. 13
B. 15
C. 17
D. 41
E. 1
41. Write the compound inequality that matches the graph below.

A. $16<x \leq 19$
B. $16<x<19$
C. $16 \geq x>19$
D. $16 \geq x<19$
E. $16 \leq x<19$
42. If $x+\frac{1}{x}=11$, then what is the value of $x^{2}+\frac{1}{x^{2}}$ ?
A. 119
B. 121
C. 123
D. 22
E. $\frac{11 \pm 3 \sqrt{13}}{2}$
43. In the picture below, $X, Y$ and $Z$ are points on the circumference of a circle of radius 7 inches and $m \angle Z X Y=30^{\circ}$. Find the length of chord $Y Z$.

A. 7.5 in
B. $7 \sqrt{2}$ in
C. $7.5 \sqrt{2}$ in
D. 7 in
E. 3.5 in
44. The cell phone Galactic Star I has a current value of $\$ 160$. With newer models coming out, the Galactic Star I's value decreases by the same percentage every year. After one year, it will be worth $\$ 128$. What will the Galactic Star I's value be after three years?
A. $\$ 76.54$
B. $\$ 92.42$
C. $\$ 84.96$
D. $\$ 81.92$
E. $\$ 77.82$
45. Caleb has a circle that has an equation of $(x-4)^{2}+(y-3)^{2}=324$. He wants to color his circle all red except for a smaller circle drawn inside the larger that has a diameter of 6 units. What is the area of the region Caleb will color red?
A. $288 \pi$ units $^{2}$
B. $318 \pi$ units $^{2}$
C. $315 \pi$ units $^{2}$
D. $312 \pi$ units $^{2}$
E. $333 \pi$ units $^{2}$
46. What is the equation of the line that passes through the points $(2,-5)$ and $(24,28)$ ?
A. $3 x+2 y=16$
B. $3 x-2 y=16$
C. $3 x-2 y=-16$
D. $-3 x+2 y=16$
E. $-3 x-2 y=-16$
47. Find the sum of the roots of the equation $28 x-2 x^{2}=6 x-12$.
A. -6
B. 6
C. 11
D. -2
E. -11
48. Two cards are drawn from a standard deck of cards, without replacement. What is the probability of drawing a king of hearts on the first draw and then an even number on the second draw?
A. $\frac{5}{663}$
B. $\frac{5}{2,652}$
C. $\frac{5}{676}$
D. $\frac{5}{1,326}$
E. $\frac{5}{169}$
49. A chemist needs to mix 20 Liters of $40 \%$ acid solution with a certain amount of $70 \%$ acid solution to create a mixture that is $50 \%$ acid. How many Liters of the $70 \%$ acid solution should the chemist use for the desired mixture?
A. 20 Liters
B. 40 Liters
C. 30 Liters
D. 10 Liters
E. 25 Liters

| 1. B | 18. C | 35. A |
| :---: | :---: | :---: |
| 2. D | 19. A | 36. A |
| 3. E | 20. C | 37. B |
| 4. D | 21. C | 38. E |
| 5. B | 22. D | 39. E |
| 6. D | 23. C | 40. A |
| 7. D | 24. C | 41. A |
| 8. B | 25. D | 42. E |
| 9. C | 26. A | 43. A |
| 10. D | 27. E | 44. D |
| 11. C | 28. E | 45. D |
| 12. B | 29. D | 46. C |
| 13. A | 30. D | 47. B |
| 14. C | 31. C | 48. C |
| 15. A | 32. B | 49. A |
| 16. C | 33. E | 50. D |
| 17. A | 34. B |  |

9. The prime factorization of 560 is $2^{4} \cdot 5 \cdot 7$. The sum of the distinct prime numbers is $2+5+7=14$. Twice 14 is equal to 28 .
10. $40 \%$ of $20 \%$ of $1 / 2$ of $400=(0.4)(0.2)(0.5)(200)=16$.
11. To find the sum of a sequence of numbers, use the formula $\frac{N(F+L)}{2}$, where $N$ equals the number of terms, $F$ equals the first term and $L$ equals the last term. Using $-8+(-6)+(-4)+(-2)+\cdots+24$, we can count there are 17 terms and substitute to find that $\frac{17(-8+24)}{2}=136$.
12. If $A=4 x-5$ and $B=7 x-6$, then $6 A-3 B=6(4 x-5)-3(7 x-6)=24 x-30-21 x+18=3 x-12$.
13. Create a proportion and solve. $\frac{1.32}{16}=\frac{x}{200}$. Cross multiply and $16 x=264$. Divide by 16 and $x=16.5$. On an item costing $\$ 200$, there will be a $\$ 16.50$ tax.
14. A triangle has $180^{\circ}$. We subtract the given $100^{\circ}$ from $180^{\circ}$ and there are $80^{\circ}$ remaining. The two angles are in a ratio of 7:9. Let $x$ be a constant and then we have the equation $7 x+9 x=80$. This gives us $16 x=80$. Dividing by 16 and we get $x=5$. The smallest angle is $7 x$, so $7(5)=35^{\circ}$.
15. $7^{x+2}=294$, so using exponent rules, $7^{x} \cdot 7^{2}=294$. Now we see that $7^{x} \cdot 49=294$ and dividing both sides by 49 and we get $7^{x}=6$. $8^{y+2}=448$, so $8^{y} \cdot 8^{2}=448$ and solving $8^{y} \cdot 64=448$ gives us $8^{y}=7$. Therefore, $7^{x}+8^{y}=6+7=13$.
16. We are given $x+\frac{1}{x}=11$ and asked to find the value of $x^{2}+\frac{1}{x^{2}}$. Square both sides of $x+\frac{1}{x}=11$ and we see that $\left(x+\frac{1}{x}\right)^{2}=11^{2} .11^{2}=121$ and $\left(x+\frac{1}{x}\right)^{2}=\left(x+\frac{1}{x}\right)\left(x+\frac{1}{x}\right)=x^{2}+\frac{x}{x}+\frac{x}{x}+\frac{1}{x^{2}}$. We can rewrite $x^{2}+\frac{x}{x}+\frac{x}{x}+\frac{1}{x^{2}}$ as $x^{2}+\frac{1}{x^{2}}+2=121$. Subtract 2 from both sides and we see that $x^{2}+\frac{1}{x^{2}}=119$.
17. A standard deck of cards has 52 cards, 4 suits with 13 different cards in each suit. The probability of drawing a king of hearts on the first draw is $\frac{1}{52}$. There are 20 even numbered cards in a standard deck, so the probability of drawing an even numbered card on the second draw, without replacement, is $\frac{20}{51}$. Thus the probability of getting a king of hearts on the first draw and then an even number, without replacement, in that order is $\frac{1}{52} \cdot \frac{20}{51}=\frac{5}{663}$.
18. Since we are mixing two amounts to get a third we have $a+b=c$. Let $a$ be equal to the 20 Liters of $40 \%$ acid, $b$ be equal to our unknown amount of $70 \%$ acid and $c$ be equal to our total amount. Now we have $20(0.4)+0.7 b=0.5(20+b)$. Our equation simplifies to $8+0.7 b=10+0.5 b$. Subtract $0.5 b$ from both sides and subtract 8 from both sides and we get $2=0.2 b$. Dividing by $0.2 \mathrm{ad} b=10$ Liters.
