TMSCA MIDDLE SCHOOL MATHEMATICS

TEST #6 ©

DECEMBER 1, 2018

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 50 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading on Scantrons and Chatsworth cards.
3. If you are using a Chatsworth or Scantron card, please follow the specific instructions given at your particular meet.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators **MAY NOT** be used on this test.
8. All problems answered correctly are worth **FIVE** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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1. $-78 + (-116) =$ _____
A. 194   B. 38   C. 42   D. -42 E. -194

2. $30.11 - 9.001 =$ _____
A. 21.1   B. 21.19   C. 21.1009   D. 20.19   E. 21.109

3. $134 \times 5\frac{1}{2} =$ _____
A. 696   B. $139\frac{1}{2}$ C. 139   D. 670   E. 737

4. $\frac{15}{4} \div \frac{4}{5} =$ _____
A. $4\frac{11}{16}$ B. 3   C. $\frac{3}{16}$ D. $2\frac{3}{8}$ E. $1\frac{5}{16}$

5. What is the GCF of the numbers 540 and 280?
A. 40   B. 70   C. 24   D. 56   E. 20

6. If $x = 18$, what is the perimeter of the rectangle?

$x + 7$

$2x - 4$
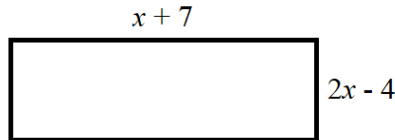
A. 266 units   B. 171 units   C. 114 units   D. 106 units   E. 96 units

7. Simplify: $2(3^2 - 2^2) + 2^3(2^3 - 2^1)$
A. 28   B. 58   C. 46   D. 62   E. 64

8. It took Michelle 1.35 hours to reel in a five-foot shark. How many minutes did it take Michelle to reel in the shark?
A. 95 minutes   B. 81 minutes   C. 21 minutes   D. 63.5 minutes   E. 93 minutes

9. What is the value of $A + B + C$, if $\frac{4}{7} = \frac{A}{35} = \frac{32}{B} = \frac{44}{C}$?
A. 161   B. 115   C. 122   D. 153   E. 147

10. If four micro-bots cost \$329.56, what is the unit rate per micro-bot?
A. \$82.39   B. \$84.29   C. \$84.19   D. \$83.79   E. \$81.59

11. 0.000015 km = _____ mm
A. 1.5   B. 150   C. 15   D. 0.15   E. 0.015

12. Lydia is buying special seasonal stamps that cost 28¢ each. How many stamps can Lydia buy with \$14.00?
A. 54   B. 36   C. 20   D. 50   E. 62

13. 99 months = _____ years
A. 7¼   B. 7½   C. 8¼   D. 8¾   E. 7¾

14. The number 620 has how many positive integral factors?
A. 12   B. 10   C. 16   D. 14   E. 8

15. If $360 = 2^x \cdot 3^y \cdot 5^z$, what is the value of $(x \cdot y)^z$?
A. 64   B. 4   C. 16   D. 6   E. 9

16. If the pattern continues, how many squares will be needed for Stage 10?

Stage 1   Stage 2   Stage 3   ...   Stage 10
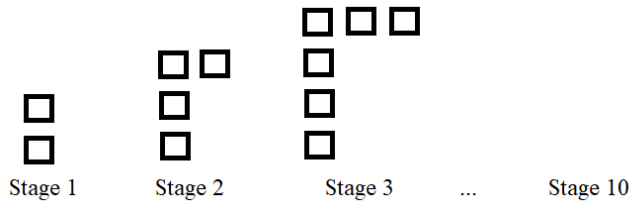
A. 48   B. 24   C. 18   D. 28   E. 20

17. If $K = 6^2 + 7^2 \cdot 2^1$, what is the remainder when K is divided by 9?
A. 7   B. 2   C. 8   D. 6   E. 5

18. Which formula gives the nth term of the sequence? $-2, -½, 1, 2½, 4, \ldots$
A. $3n - 5$ B. $\frac{5-3n}{2}$ C. $\frac{3n-7}{2}$ D. $\frac{n+1}{-2}$ E. $n - 3$

19. It takes eight boys five hours to clear a field of debris. How long would it take fourteen buys to clear the same field with the exact same amount of debris?
A. $4\frac{1}{4}$ hrs   B. $7\frac{1}{2}$ hrs   C. $2\frac{6}{7}$ hrs   D. $7\frac{3}{4}$ hrs   E. $8\frac{1}{2}$ hrs

20. The sum of three consecutive positive integers is 36. What is the product of the three integers?
A. 1,716   B. 2,184   C. 1,287   D. 1,560   E. 1,296

21. What value is 25% more than the sum of 48 and 64?
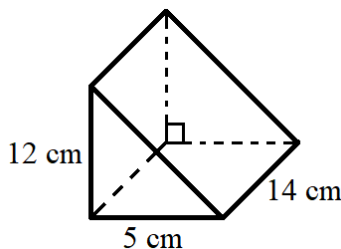A. 112   B. 136   C. 140   D. 128   E. 120

22. DL + CII – XVIII = _____ (Arabic number)
A. 624   B. 634   C. 1,134   D. 594   E. 144

23. What is the complement to the supplement of an angle measuring 137°?
A. 43° B. 47° C. 53° D. 37° E. 19°

24. What is the volume of the triangular prism below?

12 cm   14 cm   5 cm

A. 420 cm^3 B. 840 cm^3 C. 455 cm^3 D. 480 cm^3 E. 450 cm^3

25. What is the value of m in the arithmetic sequence 85, l, m, n, 33?
A. 44   B. 57   C. 59   D. 61   E. 51

26. Miguel weighed 84 pounds on Monday, 88 pounds on Tuesday, 85 pounds on Wednesday and 86 pounds on Thursday. What must Miguel weigh on Friday to have an average weight of 87 for the week?
A. 90 lbs   B. 89 lbs   C. 87 lbs   D. 88 lbs   E. 92 lbs

27. The set $\{u, v, w, x, y, z\}$ has how many improper subsets?
A. 64   B. 63   C. 2   D. 1   E. 0

28. Which of the following is a value of n, if $\frac{n}{10} - \frac{6}{n} = \frac{1}{20}$.

A. 12   B. 4   C. 8   D. 7   E. 9

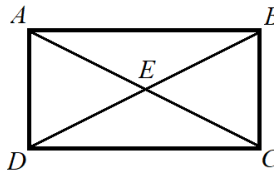29. What is the slope of the line with the equation $12x + 3y = 15$?

A. $\frac{5}{4}$ B. $-\frac{4}{5}$ C. $\frac{1}{4}$ D. -4 E. -5

30. If $f(x) = 11 - x^2$, what is the value of $\frac{1}{5}f(-6)$?

A. -5 B. $-\frac{6}{5}$ C. $\frac{47}{5}$ D. 10   E. -10

31. $ABCD$ is a rectangle. If $m\angle ABD = (4x - 5)^\circ$ and $m\angle CDB = (6x - 21)^\circ$, find $m\angle AEB$.

A. 126° B. 27° C. 63° D. 154° E. 124°

32. What is the growth rate of the exponential function $y = 78(4.11)^x$?

A. 311%   B. 411%   C. 78%   D. 320.58%   E. 780%

33. In a chemical lab experiment, the temperature inside of a beaker reached $80^\circ\,C$. What is this temperature in degrees Fahrenheit?

A. $187^\circ\,F$ B. $112^\circ\,F$ C. $176^\circ\,F$ D. $168^\circ\,F$ E. $182^\circ\,F$

34. What is the value of the discriminant of the quadratic equation $y = 3x^2 - 5x - 1$?

A. -13 B. 37   C. 2   D. 22   E. $0.8\overline{3}$

35. Which of the following is equivalent to $(5n - 12)^2$?

A. $25n^2 - 144$ B. $25n^2 + 144$ C. $25n^2 - 60n + 144$ D. $25n^2 - 120n + 144$ E. $25n^2 + 120n - 144$

36. Heidi weighs 80 pounds and is sitting on a seesaw 6 feet from the middle. Marsha weighs 64 pounds and is sitting on the opposite side of the seesaw as Heidi. How far from the middle must Marsha sit in order to balance the seesaw?

A. 9 feet   B. 10.5 feet   C. 7.5 feet   D. 8 feet   E. 8.5 feet

37. $4m^{\frac{1}{2}}$ is equivalent to which of the following?

A. $\sqrt{4m}$ B. $2\sqrt{m}$ C. $\frac{4}{m}$ D. $4\sqrt{m}$ E. $\frac{4}{m^2}$

38. Factor completely: $4w^2 - 81$

A. $(4w - 9)(4w + 9)$ B. $2(2w - 20)(2w + 20)$ C. $(2w - 9)(2w + 9)$ D. $(2w + 9)(2w + 9)$ E. $(2w - 9)(2w$ - $9)$

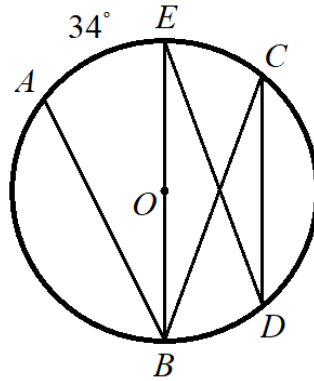39. In simplest radical form, what is the geometric mean of the numbers 18 and 16?

A. $12\sqrt{2}$ B. $4\sqrt{18}$ C. $18\sqrt{2}$ D. $\sqrt{34}$ E. $17\sqrt{2}$

40. What is the positive difference between the areas of a square with a diagonal length of $8\sqrt{2}$ units and a circle with a diameter of 6 units? Let $\pi = 3$.

A. 37 units^2 B. 42 units^2 C. 43 units^2 D. 11 units^2 E. 20 units^2

41. In the picture below, minor arc $AE = 34°$ and $m\angle EDC = 15°$. What is $m\angle ABC$?

A. $49°$ B. $24.5°$ C. $41.5°$ D. $64°$ E. $32°$

42. Which of the following is equivalent to $\log_8(5^6)$?
A. $\log_8 5 + \log_8 6$ B. $\log_8 5 \cdot \log_8 6$ C. $5\log_8 6$ D. $6\log_8 5$ E. $\log_8 30$

43. Find the value of n, if $8^{n-2} = 512$.
A. 5   B. 4   C. 3   D. 6   E. -3

44. $\sqrt{162} + \sqrt{32} = $ _____
A. $97\sqrt{2}$ B. $2\sqrt{97}$ C. $13\sqrt{2}$ D. $\sqrt{194}$ E. $6\sqrt{2}$

45. If digits may repeat, how many three-digit numbers are there for which the sum of the digits is equal to 6?
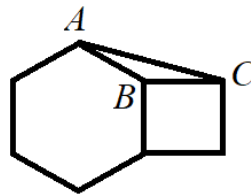A. 15   B. 18   C. 21   D. 24   E. 27

46. What is the square-root of the x-value of the solution to the system of linear equations $\begin{cases} 2x + 3y = 86 \\ 3x + 2y = 84 \end{cases}$?
A. 3   B. 4   C. 5   D. 6   E. 7

47. Rationalize the denominator of $\frac{18}{1+\sqrt{3}}$.
A. $9 + 9\sqrt{3}$ B. $-9 + 9\sqrt{3}$ C. $9 - 9\sqrt{3}$ D. $\frac{18+\sqrt{3}}{-2}$ E. $\frac{18-\sqrt{3}}{2}$

48. A regular hexagon and a square share a common side. What is $m\angle BCA$?

A. $120°$ B. $10°$ C. $15°$ D. $12°$ E. $20°$

49. Solve for n: $\left|\frac{n}{8}\right| = -16 + 35$
A. $\{\pm 93\}$ B. $\{\pm 152\}$ C. $\{-93, 152\}$ D. $\{-152, 93\}$ E. $\{93, 152\}$

50. Square $ABCD$ has a perimeter of 28 units. The radius of $\odot P$ has the same measure as the side measure of square $ABCD$. What is the equation of $\odot P$ with its center at the origin?
A. $x^2 + y^2 = 784$ B. $x^2 + y^2 = 196$ C. $x^2 + y^2 = 784$ D. $x^2 + y^2 = 112$ E. $x^2 + y^2 = 49$

1. E	18. C	35. D
2. E	19. C	36. C
3. E	20. A	37. D
4. A	21. C	38. C
5. E	22. B	39. A
6. C	23. B	40. A
7. B	24. A	41. E
8. B	25. C	42. D
9. D	26. E	43. A
10. A	27. D	44. C
11. C	28. C	45. C
12. D	29. D	46. B
13. C	30. A	47. B
14. A	31. A	48. C
15. D	32. A	49. B
16. E	33. C	50. E
17. C	34. B	

4. $\dfrac{15}{4} \div \dfrac{4}{5} = \dfrac{15}{4} \times \dfrac{5}{4} = \dfrac{75}{16} = 4\dfrac{11}{16}$.

11. Since 1 km = 1,000,000 mm, multiply 0.000015 by 1,000,000. 0.000015(1,000,000) = 15. Therefore, 0.000015 km = 15 mm.

16. One way to see the pattern is to know that each stage number requires twice that many squares. For Stage 1, 2 squares were needed. For Stage 2, four squares were needed and so on. Therefore, for Stage 10, $10 \times 2 = 20$ squares will be needed.

24. To find the area of a prism, use the formula $A = Bh$, where B is equal to the area of the base and h is equal to the height of the prism. Our base shape is a right triangle, so its area is $A = \dfrac{bh}{2} = \dfrac{5(12)}{2} = \dfrac{60}{2} = 30$. Therefore, the volume of the triangular prism is $A = Bh = 30(14) = 420 \text{ cm}^3$.

25. Since 85, l, m, n, 33 is an arithmetic sequence, we can find the common difference by $85 - 33 = 42$. Now divide 42 by 4 and the common difference is 13. Our arithmetic sequence of numbers is 85, 72, 59, 46 and 33. Therefore, the value of m is 59.

35. $(5n - 12)^2 = (5n - 12)(5n - 12) = 5n \cdot 5n - 5n(12) - 5n(12) + 12(12) = 25n^2 - 120n + 144$.

42. Since $\log_x(a^b) = b \log_x a$, $\log_8(5^6) = 6 \log_8 5$.

43. We know that $2^3 = 8$ and $512 = 2^9$. We are given $8^{n-2} = 512$, so we can rewrite as $(2^3)^{n-2} = 2^9$. By the exponent rule $(a^m)^n = a^{mn}$, we can rewrite $(2^3)^{n-2} = 2^9$ as $2^{3n-6} = 2^9$. Now, we just have to solve the equation $3n - 6 = 9$. Add 6 to both sides and $3n = 15$. Divide by 3 to both sides and $n = 5$.

45. There are 21 three-digit numbers for which the sum of the digits is equal to 6. They are 105, 114, 123, 132, 141, 150, 204, 213, 222, 231, 240, 303, 312, 321, 330, 402, 411, 420, 501, 510, and 600.

47. $\dfrac{18}{1+\sqrt{3}} \cdot \dfrac{1-\sqrt{3}}{1-\sqrt{3}} = \dfrac{18(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})} = \dfrac{18-18\sqrt{3}}{1-\sqrt{3}+\sqrt{3}-3} = \dfrac{18-18\sqrt{3}}{1-3} = \dfrac{18-18\sqrt{3}}{-2} = -9 + 9\sqrt{3}$.

48. The interior angle of a square measures $90°$ and the interior angle of a regular hexagon measures $120°$.
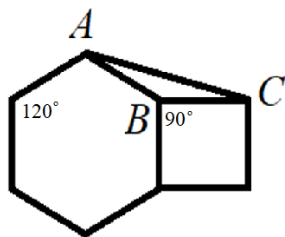
So, $m\angle ABC = 360 - 120 - 90 = 150°$. Because $AB = BC$, ΔABC is an isosceles triangle and $m\angle BAC = m\angle BCA$. A triangle has $180°$, and $180 - 150 = 30$. So, $m\angle BAC + m\angle BCA = 30$. Therefore, $m\angle BCA = 30 \div 2 = 15°$.

50. If square $ABCD$ has a perimeter of 28 units, then each side measures 7 units. If the radius of $\odot P$ has the same measure as the side measure of square $ABCD$, then its radius measures 7 units. The equation of a circle having its center at the origin is $x^2 + y^2 = r^2$, where r is the radius. So, the equation of $\odot P$ is $x^2 + y^2 = 7^2$, which is $x^2 + y^2 = 49$.