T M S C A M I D D L E S C H O O L
M A T H E M A T I C S
R E G I O N A L T E S T ©
M A R C H 2 , 2 0 1 9

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 50 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading on Scantrons and Chatsworth cards.
3. If you are using a Chatsworth or Scantron card, please follow the specific instructions given at your particular meet.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators **MAY NOT** be used on this test.
8. All problems answered correctly are worth **FIVE** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

1. $517 + (-873)$ = _____

A. 1,390 B. $-1{,}390$ C. -356 D. 304 E. -304

2. $1{,}089{,}011 - 893{,}766$ = _____ (nearest thousand)

A. 194,000 B. 196,000 C. 195,000 D. 197,000 E. 199,000

3. $\frac{5}{12} \div \frac{6}{4} \cdot \frac{2}{3}$ = _____

A. $\frac{23}{5}$ B. $\frac{5}{27}$ C. $\frac{5}{12}$ D. $\frac{5}{18}$ E. $\frac{5}{9}$

4. $328 \times 119 =$ _____

A. 39,042 B. 39,032 C. 39,112 D. 29,072 E. 39,202

5. Vanessa is calculating the product of the number of vertices and the number of edges of a hexagonal prism. What is Vanessa's product?

A. 216 B. 144 C. 148 D. 96 E. 256

6. Simplify: $\frac{1}{2}((9-(-2))^2 - 8^2 + (-19))$

A. 30 B. 38 C. 12 D. 19 E. 15

7. What is the greatest integer less than $\frac{127}{7}$?

A. 17 B. 19 C. 120 D. 18 E. 134

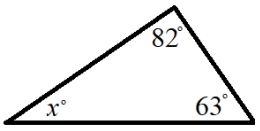
8. $\frac{1}{2}\% \neq$ _____

A. $\frac{1}{200}$ B. 0.5 C. $\frac{3}{600}$ D. 0.005 E. $\frac{7}{1400}$

9. 52 ounces = _____ cups

A. 8.5 B. 8 C. 7.5 D. 7 E. 6.5

10. What is the measure of the complement to the missing angle in the triangle?



A. 55° B. 8° C. 35° D. 27° E. 45°

11. What is the product of the reciprocals of $2\frac{2}{3}$ and $6\frac{1}{2}$?

A. $\frac{3}{37}$ B. $\frac{3}{52}$ C. $\frac{3}{8}$ D. $\frac{37}{3}$ E. $\frac{52}{7}$

12. Set *A* contains 21 elements, set *B* contains 26 elements, and the intersection of the two sets contains 7 elements. How many elements are in the union of sets *A* and *B*?

A. 45 B. 36 C. 47 D. 40 E. 42

13. Let *A* equal the sum of the reciprocals of the numbers 10 and 14. What is the reciprocal of *A*?

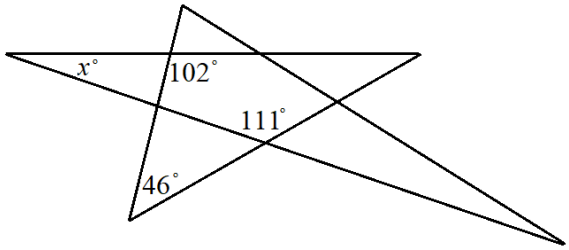
A. $\frac{35}{6}$ B. 4 C. $\frac{24}{7}$ D. 24 E. $\frac{24}{5}$

14. \$100 – 72 quarters – 344 dimes – 1,490 pennies = _____ nickels.

A. 572 B. 654 C. 728 D. 682 E. 616

15. If *A* is equal to the GCF of 50 and 64 and *B* is equal to the LCM of 24 and 82, what is the value of *A* + *B*?
A. 1,076 B. 12,084 C. 986 D. 884 E. 2,584

16. What is the value of *x* in the picture below?



A. 37 B. 17 C. 27 D. 47 E. 7

17. Cassy drew a $18cm \times 13\ cm$ rectangle and a square with a perimeter of 64 cm. How much greater is the area of the square than the area of the rectangle?
A. 26 cm^2 B. 18 cm^2 C. 17 cm^2 D. 2 cm^2 E. 22 cm^2

18. What is the remainder when 176,980 is divided by 15?
A. 14 B. 10 C. 12 D. 4 E. 6

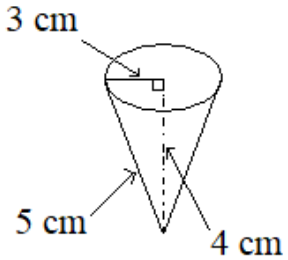
19. MDXLIX – DCCLXXVIII = _____ (Arabic number)
A. 778 B. 229 C. 458 D. 771 E. 783

20. What is the sum of the distinct prime factors of 4,312?
A. 33 B. 31 C. 26 D. 22 E. 20

21. $16_7 + 143_5 - 54_6 =$ _____$_8$
A. 27 B. 34 C. 33 D. 41 E. 43

22. If $0.5\overline{7}$ can be written as $\frac{A}{B}$ in lowest terms, then what is the value of 2*A* – *B*?
A. 7 B. 11 C. 19 D. 2 E. 9

23. William only wants to fill his cone half-full of sand. How much sand will William need? Let $\pi = 3$.

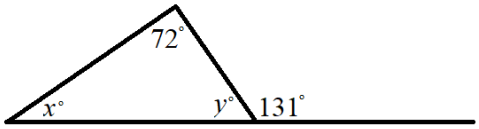


A. 30 cm^3 B. 16 cm^3 C. 22.5 cm^3 D. 24 cm^3 E. 18 cm^3

24. The length of a rectangle is 4*x* – 3 and its width is 2*x* + 5. What is the value of *x* if the perimeter of the rectangle is 76 units?
A. $12.\overline{3}$ B. 4 C. 6 D. 7 E. $8.\overline{3}$

25. Wyatt use 24 ounces of meat to make 8 hamburgers for his friends. How many pounds of meat does Wyatt need to make 72 hamburgers for his school fundraiser dinner?
A. 108 pounds B. 20.25 pounds C. 9 pounds D. 13.5 pounds E. 6.75 pounds

26. What is the value of *x* in the picture?



A. 49 B. 79 C. 89 D. 68 E. 59

27. If $A = \{a, b, c, d, e, f\}$ and $B = \{c, d, e, g, h, f\}$, how many subsets does $A \cap B$ have?
A. 15 B. 32 C. 64 D. 16 E. 31

28. The 1st term of a geometric sequence is ½ and the common ratio is 6. What is the 4th term of the sequence?
A. 162 B. 124 C. 108 D. 1,296 E. 116

29. Solve: $\log x + \log(x + 48) = 2$
A. $\{2, -50\}$ B. $\{2,50\}$ C. $\{-2\}$ D. $\{2\}$ E. $\{-50\}$

30. Using any combination of quarters, dimes, nickels and pennies, how many different ways can 30¢ be made?
A. 18 B. 24 C. 16 D. 20 E. 28

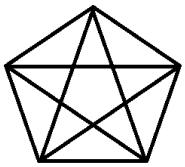
31. If the tax rate is 8%, what is Marcy's total bill if she buys a lip gloss for $4.00, some fruit for $2.50 and two pairs of shoes for $34.00 each?
A. $46.46 B. $43.74 C. $129.60 D. $80.26 E. $80.46

32. What is the volume of a sphere with a radius of 8 cm? Let $\pi = 3$.
A. 1,024 cm^3 B. 1,536 cm^3 C. 3,072 cm^3 D. 2,048 cm^3 E. 768 cm^3

33. *C* is the midpoint of $\overline{AB}$. If *A* has coordinates (45, 17) and *C* has coordinates (67, 64), what are the coordinates of *B*?
A. (56, 47) B. (31, 111) C. (89, 111) D. (56, 40.5) E. (89, 98)

34. What is the equation of the axis of symmetry of the graph of the quadratic equation $y = 4x^2 + 16x - 9$ after it is translated to the right eight units?
A. $x = 2$ B. $x = -2$ C. $x = -8$ D. $x = 6$ E. $x = 10$

35. How many triangles can be found in the picture below?



A. 48 B. 35 C. 27 D. 30 E. 42

36. On linear function g, $g(-4) = -4$ and $g(8) = 5$. Find $g(16)$.
A. 11 B. 12 C. 9 D. 14 E. 10

37. How many positive two-digit integers have an odd product of its digits?
A. 19 B. 21 C. 25 D. 27 E. 32

38. $\left(\sqrt{80} + \sqrt{40}\right)\left(2\sqrt{20} - 2\sqrt{10}\right)$ = _____
A. 40 B. $2\sqrt{5} + 4\sqrt{2}$ C. 120 D. $120 - 80\sqrt{2}$ E. 60

39. Simplify: $\left(\frac{1}{8b^4}\right)^{-1} \cdot \left(\frac{(4ab^2)^2}{2a^3b}\right)^{-1}$

A. ab B. $\frac{1}{ab}$ C. $\frac{a}{b}$ D. $\frac{b}{a}$ E. $\frac{2a}{b}$

40. If $\pi = 3$, what is the circumference of the circle with equation $8 + x^2 + y^2 - 8x - 6y + 4 = -4$?
A. 15 units B. 24 units C. 21 units D. 20 units E. 18 units

41. What is the perimeter of an equilateral triangle with an altitude of 12 cm?
A. $27\sqrt{3}$ cm B. $18\sqrt{3}$ cm C. $36\sqrt{3}$ cm D. $30\sqrt{3}$ cm E. $24\sqrt{3}$ cm

42. *M* and *N* are the solutions of the equation $\frac{|2x-4|}{6} = 7$. What is the value of $(|N + M|)^2$?
A. 25 B. 9 C. 16 D. 36 E. 4

43. What is the smallest positive integer *n* for which $16^{20} < 8^n$?
A. 7 B. 17 C. 27 D. 21 E. 10

44. If *m* and *n* are positive integers, what is the value of *m* in the equation $m!\,n! = m! + n!$?
A. 6 B. 4 C. 3 D. 2 E. 8

45. A boat can travel 16 miles upstream in 2 hours. The same boat can travel 48 miles downstream in 3 hours. What is the speed of the current?
A. 2 mph B. 6 mph C. 3 mph D. 8 mph E. 4 mph

46. Rationalize the denominator: $\frac{2+\sqrt{8}}{4-\sqrt{8}}$ = _____

A. $\frac{4+\sqrt{2}}{8}$ B. $\frac{2+\sqrt{2}}{2}$ C. $\frac{4+3\sqrt{2}}{2}$ D. $\frac{8+3\sqrt{2}}{4}$ E. $\frac{4+3\sqrt{2}}{4}$

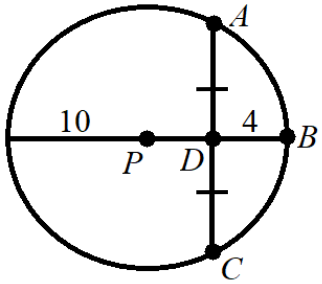
47. What is the sum of the roots of the cubic equation $9x^3 - 6x^2 - 3x - 2 = 0$?
A. ⅓ B. ¾ C. ½ D. ⅔ E. ¼

48. $\log_7\left(\sqrt[3]{ab}\right)$ = _____.

A. $\frac{\log_7 a}{3} + \frac{\log_7 b}{3}$ B. $\frac{\log_7 a}{3} - \frac{\log_7 b}{3}$ C. $3\left(\frac{\log_7 a}{7} + \frac{\log_7 b}{7}\right)$ D. $\log_7(ab)^3$ E. $\frac{\log_7(ab)^3}{3}$

49. An art piece was donated at the local museum. The art piece is worth $500 and increases at a rate of 20% each year. What will the art piece be worth after 2 years?
A. $1,200 B. $720 C. $680 D. $690 E. $740

50. Using the picture below, $\odot P$ has a radius of 10 inches and *DB* = 4 inches. What is the measure of chord *AC*?



A. 18.5 inches B. 16 inches C. 18 inches D. 12 inches E. 20 inches

2018 – 2019 TMSCA Middle School Mathematics Regional Test Answer Key

1. C
2. C
3. B
4. B
5. A
6. D
7. D
8. B
9. E
10. A
11. B
12. D
13. A
14. B
15. C
16. A
17. E
18. B
19. D
20. E
21. C
22. A
23. E
24. C
25. D
26. E
27. D
28. C
29. D
30. A
31. E
32. D
33. C
34. D
35. B
36. A
37. C
38. A
39. A
40. E
41. E
42. C
43. C
44. D
45. E
46. C
47. D
48. A
49. B
50. B

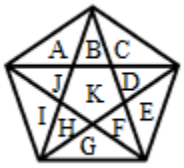
2018-2019 TMSCA Middle School Mathematics Regional Test Selected Answers

20. $4{,}312 = 2^3 \cdot 7^2 \cdot 11$, therefore 2 + 7 + 11 = 20.

30. Let q = quarters, d = dimes, n = nickels and p = pennies. Make a list to show the combinations. They are: $q + n$, $q + 5p$, $3d$, $2d + 2n$, $2d + n + 5p$, $2d + 10p$, $d + 4n$, $d + 3n + 5p$, $d + 2n + 10p$, $d + n + 15p$, $d + 20p$, $6n$, $5n + 5p$, $4n + 10p$, $3n + 15p$, $2n + 20p$, $n + 25p$ and $30p$. Therefore, there are 18 combinations.

31. 4 + 2.5 + 34 + 34 = 74.50. The tax rate is 8%, so 74.5(1.08) = \$80.46 as the total bill.

35. Label the figure as shown. There are 10 triangles using 1 section, which are A, B, C, D, E, F, G, H, I and J.



There are 10 triangles using 2 sections, which are AB, BC, CD, DE, EF, FG, GH, HI, IJ and JA. There are 10 triangles using 3 sections, which are ABC, JKD, CDE, BKF, GFE, HKD, GHI, JKF, AJI and HKB. There are 0 triangles using 4 sections. There are 5 triangles using 5 sections, which are FKJBA, DKJHI, BKHFG, JKDEF and HKBCD. Therefore, there are 10 + 10 + 10 + 5 = 35 triangles in the figure.

38. $\left(\sqrt{80}+\sqrt{40}\right)\left(2\sqrt{20}-2\sqrt{10}\right) = \left(4\sqrt{5}+2\sqrt{10}\right)\left(4\sqrt{5}-2\sqrt{10}\right) = 80 + 40\sqrt{2} - 40\sqrt{2} - 40 = 40.$

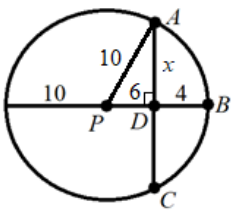
39. $\left(\frac{1}{8b^4}\right)^{-1} \cdot \left(\frac{\left(4ab^2\right)^2}{2a^3b}\right)^{-1} = (8b^4) \cdot \left(\frac{16a^2b^4}{2a^3b}\right)^{-1} = (8b^4) \cdot \left(\frac{8b^3}{a}\right)^{-1} = (8b^4)\left(\frac{a}{8b^3}\right) = \frac{8ab^4}{8b^3} = ab.$

43. $16 = 2^4$ and $8 = 2^3$, so $16^{20} < 8^n$ can be rewritten as $(2^4)^{20} < (2^3)^n$. This can be rewritten as $2^{80} < 2^{3n}$. Now we only need to solve the inequality $80 < 3n$. Divide both sides by 3 and we get $26\frac{2}{3} < n$. Therefore, the smallest positive integer for n is 27.

45. Let b equal the speed of the boat and c equal the speed of the current. We create the equation $16 = 2(b - c)$ and $48 = 3(b + c)$. Divide both side of the first equation by 2 and we get $8 = b - c$. Divide both side of the second equation by 3 and we get $b + c = 16$. We now have the system $\begin{cases} b - c = 8 \\ b + c = 16 \end{cases}$. Add these equations together and we get $2b = 24$, and solving gives $b = 12$ mph. Since the boat has a speed of 12 mph, substitute into the equation $b + c = 16$ and get $12 + c = 16$. Solve this to get $c = 4$. The speed of the current is 4 mph.

49. This is an example of an exponential growth function, $y = ab^x$, where a is the initial amount, b is our growth factor and x is time. The growth factor is equal to $1 + r$, where r is the rate. From our problem, we have the equation $y = 500(1.2)^2$, and $y = 500(1.2)^2 = 500(1.44) = 720$. Therefore, after 2 years, the art piece will be worth \$720.

50.



Draw radius $\overline{AP}$, which is also 10 inches. Since the radius is 10 inches, $PD = 10 - 4 = 6$ inches. We now have a right triangle and we need to find x, which is AD. Using the Pythagorean Theorem, $x = \sqrt{10^2 - 6^2} = 8$ inches. Since AD is only half of AC, we double the length of AD to get 16 inches. Therefore, AC = 16 inches.