



**TMSCA MIDDLE SCHOOL
MATHEMATICS
STATE TEST ©
APRIL 22, 2017**

GENERAL DIRECTIONS

1. About this test:
 - A. You will be given 40 minutes to take this test.
 - B. There are 50 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading on Scantrons and Chatsworth cards.
3. If you are using a Chatsworth or Scantron card, please follow the specific instructions given at your particular meet.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators **MAY NOT** be used on this test.
8. All problems answered correctly are worth **FIVE** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2016 – 2017 TMSCA Middle School State Mathematics Championship Test

1. Which expression below produces the smallest value?

- A. $\left(\frac{1}{2}\right)^3$ B. $\left(\frac{3}{5}\right)^2$ C. $\frac{1}{3} + \frac{1}{3}$ D. $\left(\frac{3}{4} - \frac{1}{2}\right)^2$ E. $\left(\frac{2}{3} - \frac{1}{3}\right)^2$

2. Let $A = \frac{3}{4}$, $B = \frac{1}{12}$, and $C = \frac{A+B}{2}$. What is the sum of the numerator and denominator of C ?

- A. 21 B. 88 C. 12 D. 13 E. 17

3. How many sides does a regular polygon have if the measure of its exterior angle is 15° ?

- A. 20 B. 21 C. 22 D. 23 E. 24

4. Randy's lunch bill came out to be \$28.50. Randy is going to leave an 18% tip plus what change he has in his pocket. If Randy has \$0.71 in change in his pocket, how much will Randy's total tip be?

- A. \$5.13 B. \$33.63 C. \$34.34 D. \$5.84 E. \$29.21

5. What is the sum of the eight smallest distinct positive integer multiples of 7?

- A. 248 B. 246 C. 245 D. 308 E. 252

6. What is the positive difference between 150% of 3,100 and 110% of 1,900?

- A. 2,840 B. 2,560 C. 2,220 D. 2,750 E. 2,180

7. Find the value of the reciprocal of the number $6.\overline{41}$.

- A. $\frac{90}{581}$ B. $\frac{41}{90}$ C. $\frac{14}{33}$ D. $\frac{99}{127}$ E. $\frac{99}{635}$

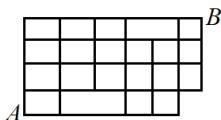
8. If $a = 1, b = 2, c = 3, \dots, z = 26$, what is the product of the letters of the word *radical*?

- A. 23,418 B. 23,824 C. 7,776 D. 23,328 E. 24,136

9. If $a \uparrow b = \frac{a}{2} + 3b$ and $m \Downarrow n = mn^2$, then find the sum of $(7 \uparrow 1.5)$ and $\left(\frac{1}{8} \Downarrow 4\right)$.

- A. 12.5 B. 8.5 C. 8 D. 10 E. 12

10. Moving only to the right or upwards, how many paths are there from A to B ?



- A. 125 B. 135 C. 145 D. 155 E. 130

11. For a school project, Lucy had to measure the thickness of her cat's fur using a microscope. She measured one hair to have a diameter of 0.0000000000078 cm. For fun, Lucy measured her dog's fur and found one hair to be four times as thick as her cat's. What is the radius of Lucy's dog's hair?

- A. 7.8×10^{-13} B. 3.9×10^{-13} C. 1.95×10^{-12} D. 2.65×10^{-12} E. 1.56×10^{-12}

12. The ratio $\frac{3}{5}:6$ is equal to $4:M$. What is the value of M ?

- A. 16 B. 32 C. 128 D. 72 E. 64

13. Let set $A = \{16, 24, 18, 30, 16, 10\}$. If each element in A were increased by the arithmetic mean of the elements in A to create set B , what is the value of the upper-quartile of B ?

- A. 43 B. 46 C. 35 D. 38 E. 45

14. $45^2 - 23^2 =$ _____ (Roman numeral)

- A. MXDXCVI B. MCDXCVI C. MMCCXXII D. MMCDXCVI E. MCDCVI

15. \overline{BD} bisects $\angle ABC$, \overline{BE} bisects $\angle DBC$, \overline{BF} bisects $\angle ABD$, and \overline{BG} bisects $\angle ABF$. What is the measure of $\angle FBC$, if the measure of $\angle ABG$ is equal to the measure of the complement of an angle measuring 77.5° ?

- A. 50° B. 125° C. 102.5° D. 75° E. 65°

16. During a magic show, a group of eight audience members are asked to walk on the stage. If five audience members of the group are asked to participate in a magic trick, how many combinations are possible?

- A. 72 B. 24 C. 56 D. 42 E. 36

17. Simplify: $0.375 + \left(\frac{2}{5} - 0.75 \cdot \frac{4}{5}\right) \div \frac{1}{4} + \frac{1}{4} \div 0.5$

- A. $\frac{3}{40}$ B. $\frac{1}{8}$ C. $\frac{3}{16}$ D. $\frac{3}{32}$ E. $\frac{5}{16}$

18. The current record for the Vikings baseball team is 22 wins with 8 losses. The Vikings have 20 remaining games. What will the Vikings' overall winning percentage be if they win 4 out of every 5 games they have remaining?

- A. 44% B. 52% C. 76% D. 74% E. 82%

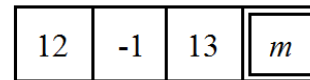
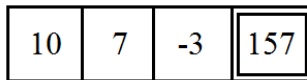
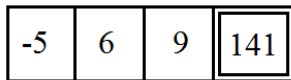
19. Sean has six brothers and five sisters. Sean's sister Harriet has x sisters and y brothers. Find the value of xy .

- A. 30 B. 25 C. 42 D. 28 E. 35

20. What is the sum of the number of edges, faces and vertices of a hexagonal prism and dodecagonal prism?

- A. 108 B. 112 C. 116 D. 124 E. 96

21. Use the examples below to find the value of m .



- A. 316 B. 315 C. 314 D. 313 E. 312

22. Thomas recorded a video for a school project that was 1.3 hours long. The assignment said the recording could only be 75 minutes long. How many minutes must Thomas shorten his video by to meet the requirements of the assignment?

- A. 3 minutes B. 8 minutes C. 11 minutes D. 5 minutes E. 1 minute

23. A $3 \text{ in} \times 3 \text{ in} \times 3 \text{ in}$ cube of gold is worth \$1,350. How much is a $5 \text{ in} \times 5 \text{ in} \times 5 \text{ in}$ cube of gold worth?

- A. \$6,750 B. \$4,500 C. \$5,250 D. \$6,500 E. \$6,250

24. What is the x -intercept of the line that passes through the points $(-2, 8)$ and $(6, -12)$?

- A. 0.6 B. 0.8 C. 1.0 D. 1.2 E. 1.4

25. Billy is taking a trip. Billy's car gets 18 miles per gallon when he drives a constant rate of 65 mph and gets 22 miles per gallon when he drives 55 mph. If the trip is 198 miles and the cost of gas is \$2.10 per gallon, how much will Billy save by keeping a constant rate of 55 mph instead of 65 mph?

- A. \$4.00 B. \$4.10 C. \$4.20 D. \$4.30 E. \$4.40

26. Rachel will flip a coin to determine who will win the tie-breaker of the game she is playing with a friend. The function $f(x) = -2x^2 + 4x + 3$ models the path of the coin, where x is the time, in seconds, the coin is in the air and $f(x)$ is the height, in feet, of the coin. After how many seconds will the coin reach its maximum height?

- A. 1 second B. $\frac{1}{2}$ second C. 2 seconds D. $\frac{3}{4}$ second E. $1\frac{1}{2}$ seconds

27. A square is inscribed inside a circle. If the perimeter of the square is 24 cm, what is the area of the circle?

- A. $18\pi \text{ cm}^2$ B. $144\pi \text{ cm}^2$ C. $72\pi \text{ cm}^2$ D. $36\pi \text{ cm}^2$ E. $12\pi \text{ cm}^2$

28. *Yards for Less* rents lawn mowers for \$12.00 plus \$2.50 per hour. Nick can spend no more than \$36.00 to mow his front and back yards. What is the maximum amount of hours, as a whole number, that Nick can rent the lawn mower?

- A. 7 hours B. 9 hours C. 10 hours D. 8 hours E. 6 hours

29. $34_7 + 102_6 + 77_9 = 111_2 \times \text{_____}_4$

- A. 121 B. 103 C. 123 D. 131 E. 113

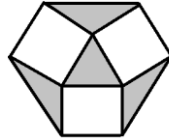
30. What is the sum of the 61st and 83rd terms of the arithmetic sequence? -24, -18, -12, -6, ...

- A. 776 B. 1,140 C. 672 D. 804 E. 786

31. Students in Mr. Chu’s calculus class took two tests. If 27% of the students passed both tests and 60% passed the first test, what is the probability that a student who passed the first test also passed the second test?

- A. 45% B. 42% C. 36% D. 48% E. 16.2%

32. The picture below consists of one equilateral triangle, three congruent squares and three other congruent triangles. If the side length of a square is 4 inches, what is the combined perimeter of all the triangles?



- A. $36 + 8\sqrt{3}$ inches B. $36 + 3\sqrt{3}$ inches C. $36 + 6\sqrt{3}$ inches D. $36 + 16\sqrt{3}$ inches E. $36 + 12\sqrt{3}$ inches

33. Calculate the discriminant of the quadratic equation $(5x - 6)(3x + 7) = 0$.

- A. -2,231 B. 2,554 C. -2,486 D. 1,828 E. 2,809

34. Sphere *A* has a diameter of 12 and sphere *B* has a diameter of 18. What is the ratio of the volume of sphere *A* to the volume of sphere *B*?

- A. 2:3 B. 8:27 C. 1:27 D. 3:16 E. 4:9

35. Let *A* equal the sum of the roots and *B* be the product of the roots of the equation $\frac{x}{x+1} = \frac{6}{x+5}$. Find the value of *A* - *B*.

- A. 7 B. -5 C. -6 D. -1 E. 1

36. A rectangle has a length of 20 cm and a width of 12 cm. If the length is increased by 20% and the width is increased by 50%, then the area of the rectangle has a percent increase of which of the following?

- A. 80% B. 75% C. 35% D. 70% E. 60%

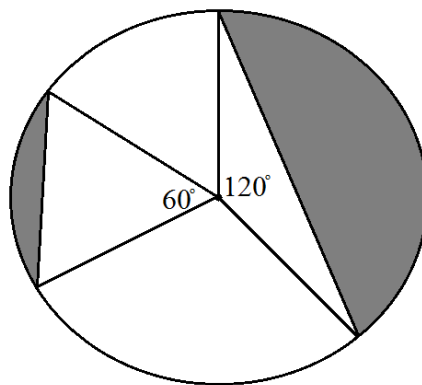
37. The ratio of the angles in a pentagon is 14:24:28:30:39. Let the difference of the largest angle and the smallest angle be *D*. Find the supplement of *D*.

- A. 90° B. 75° C. 60° D. 100° E. 80°

38. If $x + \frac{1}{x} = 4$ and $x^4 + \frac{1}{x^4} = y$, then simplify $\sqrt{y - 6}$.

- A. $3\sqrt{22}$ B. $2\sqrt{22}$ C. $3\sqrt{43}$ D. $2\sqrt{19}$ E. $2\sqrt{47}$

39. Find the total area of the shaded regions in the circle below that has a diameter of 36 cm.



- A. $108\pi - 162\sqrt{3} \text{ cm}^2$ B. $154\pi - 9\sqrt{3} \text{ cm}^2$ C. $648\pi - 81\sqrt{3} \text{ cm}^2$ D. $162\pi - 81\sqrt{3} \text{ cm}^2$ E. $162\pi - 162\sqrt{3} \text{ cm}^2$

40. [-26, 6] is the solution set for which of the following inequalities?

- A. $1 \leq 3 + \frac{2}{3}x \leq 7$ B. $-6 \leq 3 + \frac{1}{3}x \leq 5$ C. $-12 \leq \frac{1}{2}x + 1 \leq 4$ D. $2 \leq 6 + \frac{3}{4}x \leq 12$ E. $-7 \leq \frac{5}{4}x - 1 \leq -1$

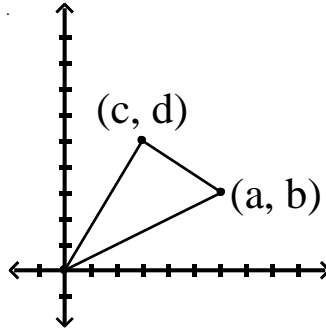
41. Let $A = 4^8 - 1$. One divisor of *A* is 257. What is the sum of all the divisors of *A* greater than 50, but less than 100?

- A. 136 B. 148 C. 142 D. 132 E. 138

42. If $f(x) = x^2 + 5x - 7$, $g(x) = 11 - x$ and $h(x) = x^2 - 16$, find $f(a + 5) - g(2a - 9) + h(2a - 3)$.
 A. $5a^2 + 5a + 27$ B. $5a^2 + 15a + 56$ C. $5a^2 + 13a - 7$ D. $5a^2 + 15a + 43$ E. $5a^2 + 5a + 16$

43. What is the area of a rhombus with side length 4 cm and one interior angle measure equal to 60 degrees?
 A. $16\sqrt{3}$ cm² B. $8\sqrt{3}$ cm² C. 8 cm² D. $2\sqrt{3}$ cm² E. $4\sqrt{3}$ cm²

44. What is the area of the triangle below?



A. $\frac{a(c-d)}{2}$ B. $\frac{ab+cd}{2}$ C. $\frac{ab-cd}{2}$ D. $\frac{ad-bc}{2}$ E. $\frac{ac+bd}{2}$

45. The graph of $x^2 + y^2 - 22x + 28y = -61$ is a circle. If the radius of the circle is dilated by a scale factor of 2.5, what is the new circumference of the circle?

A. 60π units B. 64π units C. 80π units D. 96π units E. 108π units

46. Let A equal the mean absolute deviation of the set of numbers {16, 20, 24, 28} and let B equal the mean absolute deviation for the set of numbers {27, 36, 45, 54}. What is the geometric mean of A and B ?

A. $2\sqrt{6}$ B. $4\sqrt{6}$ C. $6\sqrt{2}$ D. 4 E. 6

47. Simplify: $\left(\sqrt[3]{\sqrt{64\sqrt{64}}}\right)^2 + (\sqrt{2})^4$

A. 12 B. 6 C. 24 D. 32 E. 16

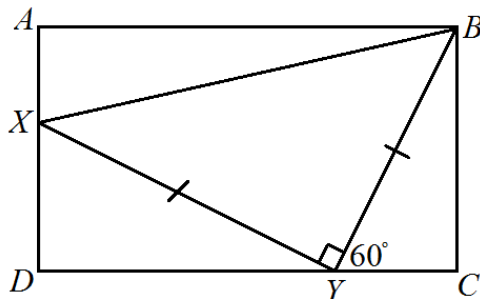
48. The coordinates of the vertices of a parallelogram are (1, 6), (-3, -2), (8, -2) and (x, y) . What is the sum of the distinct possible values for x ?

A. 8 B. 2 C. 4 D. 6 E. 10

49. f is a function defined as $f(x) + \frac{1}{x} \cdot f\left(\frac{2}{x}\right) = 4$. Find the value of $f(3)$.

A. $5\frac{1}{3}$ B. $5\frac{2}{3}$ C. $5\frac{3}{8}$ D. $5\frac{1}{8}$ E. $5\frac{5}{8}$

50. In the rectangle below, $YB = 4$ inches. Find the value of mn , if $AB = m$ and $AX = n$?



A. 12 B. 8 C. 6 D. $4 - 4\sqrt{3}$ E. $4 - 4\sqrt{6}$

2016 – 2017 TMSCA Middle School Mathematics State Championship Test Answer Key

1. D	18. C	35. A
2. E	19. D	36. A
3. E	20. B	37. E
4. D	21. D	38. E
5. E	22. A	39. E
6. B	23. E	40. C
7. E	24. D	41. A
8. D	25. C	42. E
9. D	26. A	43. A
10. C	27. A	44. D
11. E	28. B	45. C
12. E	29. B	46. E
13. A	30. D	47. A
14. B	31. A	48. D
15. D	32. E	49. A
16. C	33. E	50. B
17. A	34. B	

5. The 8 multiples of 7 are 7, 14, 21, 28, 35, 42, 49 and 56. Thus, $7 + 14 + 21 + 28 + 35 + 42 + 49 + 56 = 252$.

19. Sean has six brothers and five sisters. If Harriet is Sean's sister, then Harriet has 4 sisters and 7 brothers. Therefore, $4 \cdot 7 = 28$.

23. A $3 \text{ in} \times 3 \text{ in} \times 3 \text{ in}$ cube is made up of 27 $1 \text{ in} \times 1 \text{ in} \times 1 \text{ in}$ smaller cubes. This means that each smaller cube is worth $\$1,350 \div 27 = \50 per small cube. A $5 \text{ in} \times 5 \text{ in} \times 5 \text{ in}$ cube is made up of 125 smaller cubes, so $125(50) = \$6,250$.

31. If 27% of the students passed both tests and 60% passed the first test, then the probability that a student who passed the first test also passed the second test is $\frac{27}{60} = 0.45 = 45\%$.

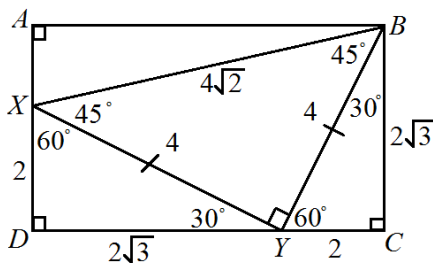
33. $(5x - 6)(3x + 7) = 15x^2 + 35x - 18x - 42 = 15x^2 + 17x - 42$. To find the discriminant of a quadratic equation $Ax^2 + Bx + C$, use $B^2 - 4AC$. Using our equation, $17^2 - 4(15)(-42) = 2,809$.

49. We are given the function $f(x) + \frac{1}{x} \cdot f\left(\frac{2}{x}\right) = 4$, so $f(3) + \frac{1}{3} \cdot f\left(\frac{2}{3}\right) = 4$. We do not know what $f\left(\frac{2}{3}\right)$ is, so let's substitute in and $f\left(\frac{2}{3}\right) + \frac{1}{\frac{2}{3}} \cdot f\left(\frac{2}{\frac{2}{3}}\right) = f\left(\frac{2}{3}\right) + \frac{3}{2} \cdot f(3) = 4$. We can now create a system if we

let $a = f(3)$ and $b = f\left(\frac{2}{3}\right)$. Our system is then, $\begin{cases} a + \frac{1}{3}b = 4 \\ b + \frac{3}{2}a = 4 \end{cases} = \begin{cases} a + \frac{1}{3}b = 4 \\ \frac{3}{2}a + b = 4 \end{cases}$. We are trying to find $f(3)$,

so we must eliminate b . Multiply the first equation by 3 and $3\left(a + \frac{1}{3}b = 4\right) = 3a + b = 12$. Subtract the second equation from the first and $3a + b - \left(\frac{3}{2}a + b\right) = 12 - 4$ and we have $\frac{3}{2}a = 8$. Multiply both sides by $\frac{2}{3}$ and $a = \frac{16}{3} = 5\frac{1}{3} = f(3)$.

50. With the given information, we can find the missing measure as such,



From this, we can see that $AB = 2 + 2\sqrt{3} = m$, and that $n = AX = 2\sqrt{3} - 2$. Therefore, $mn = (2 + 2\sqrt{3})(2\sqrt{3} - 2) = 4\sqrt{3} - 4 + 12 - 4\sqrt{3} = 8$