



**TMSCA MIDDLE SCHOOL  
MATHEMATICS  
STATE TEST ©  
APRIL 27, 2019**

**GENERAL DIRECTIONS**

- About this test:
  - You will be given 40 minutes to take this test.
  - There are 50 problems on this test.
- All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading on Scantrons and Chatsworth cards.
- If you are using a Chatsworth or Scantron card, please follow the specific instructions given at your particular meet.
- You may write anywhere on the test itself. You must write only answers on the answer sheet.
- You may use additional scratch paper provided by the contest director.
- All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
- Calculators **MAY NOT** be used on this test.
- All problems answered correctly are worth **FIVE** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
- In case of ties, percent accuracy will be used as a tie breaker.



2018 – 2019 TMSCA Middle School State Mathematics Championship Test

1.  $987 + 321 - 1,008 =$  \_\_\_\_\_

- A. -200                      B. -21                      C. 100                      D. 300                      E. 21

2.  $3\frac{2}{5} - 7\frac{1}{4} =$  \_\_\_\_\_

- A.  $-3\frac{17}{20}$                       B.  $-3\frac{11}{20}$                       C.  $-3\frac{1}{9}$                       D.  $-3\frac{7}{9}$                       E.  $-4\frac{11}{20}$

3.  $1,048 \div \frac{4}{5} =$  \_\_\_\_\_ (nearest hundred)

- A. 1,200                      B. 840                      C. 800                      D. 900                      E. 1,300

4.  $\frac{5}{6} \times \frac{2}{5} \times \frac{12}{5} \times \frac{10}{3} =$  \_\_\_\_\_

- A.  $2.1\bar{6}$                       B.  $2.\bar{6}$                       C.  $2.\bar{67}$                       D.  $2.6\bar{7}$                       E.  $2.\bar{7}$

5. Simplify:  $\frac{7}{8}(12 - 2^3)^2 \div \frac{1}{2}(-8 + 10)$

- A. 14                      B. 28                      C. -28                      D. 40                      E. 56

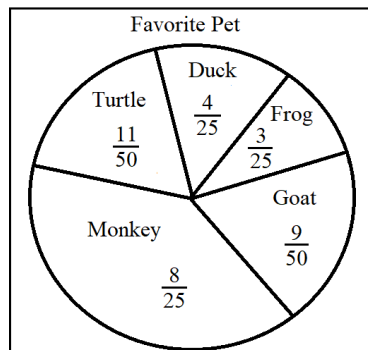
6.  $\frac{3}{16} + \frac{3}{8} =$  \_\_\_\_\_%

- A. 56.25                      B. 56.75                      C. 58.25                      D. 58.75                      E. 52.75

7.  $24\frac{1}{3} \neq$  \_\_\_\_\_

- A.  $24.\bar{3}$                       B.  $\frac{73}{3}$                       C.  $243.\bar{3}\%$                       D.  $2,433.\bar{3}\%$                       E.  $\frac{219}{9}$

8. 250 students were asked what was their favorite pet. The results are shown in the graph. How many more students chose a monkey as their favorite pet than a turtle?



- A. 30                      B. 25                      C. 17                      D. 10                      E. 27

9. If  $A = 1, B = 2, C = 3, \dots, Y = 25$  and  $Z = 26$ , what is the positive difference between the sums of the values of the letters of the word *VIOLET* and *FUCHSIA*?

- A. 18                      B. 16                      C. 23                      D. 21                      E. 17

10. If  $A + B + C = 496$ , what is the value of  $C$  if  $A = \text{GCF of } 200 \text{ and } 160$ , and  $B = \text{LCM of } 38 \text{ and } 12$ ?

- A. 234                      B. 228                      C. 214                      D. 342                      E. 84

11. What is the sum of the mean and median of the set of numbers  $\{14, 22, 34, 30, 25\}$ ?

- A. 50                      B. 59                      C. 45                      D. 48                      E. 55

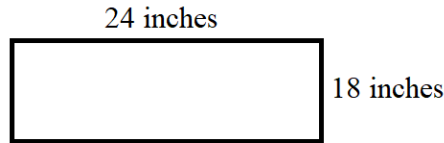
12. 3.5 gallons = \_\_\_\_\_ cubic inches

- A. 808.5                      B. 448                      C. 448.5                      D. 1,204.5                      E. 924.5

13. What is the measure of the supplement to the complement of an angle measuring  $14^\circ$ ?  
 A.  $166^\circ$                       B.  $76^\circ$                       C.  $118^\circ$                       D.  $104^\circ$                       E.  $114^\circ$

14. Solve:  $-\frac{156}{w} \geq -12$   
 A.  $w \leq 1,872$                       B.  $w \leq 13$                       C.  $w \geq 13$                       D.  $w \leq -1,872$                       E.  $w \geq -13$

15. Using the rectangle below, what is the new perimeter of the rectangle if the length and width are increased by 25%?



A. 105 inches                      B. 84 inches                      C. 63 inches                      D. 126 inches                      E. 108 inches

16. The number 600 has six odd positive integral divisors. What is the sum of these odd integral divisors?  
 A. 136                      B. 124                      C. 138                      D. 126                      E. 130

17.  $65,981 - 63,769 =$  \_\_\_\_\_ (Roman numeral)  
 A. MMCCXXII                      B. MMCCVII                      C. MMDCCXII                      D. MMDCCVII                      E. MMCCXII

18. If  $a \heartsuit b = \sqrt{ab} - a + b$ , then what is the value of  $(-8) \heartsuit (-18)$ ?  
 A.  $-2$                       B. 6                      C. 2                      D.  $-6$                       E. 20

19.  $1.2 \text{ km} - 10,000 \text{ cm} - 120,000 \text{ mm} =$  \_\_\_\_\_ meters  
 A. 80                      B. 10,880                      C. 1,088                      D. 180                      E. 980

20. Line  $L$  passes through the points  $(8, 4)$  and  $(-2, 2)$ . What is the slope of any line perpendicular to line  $L$ ?  
 A.  $-10$                       B.  $-5$                       C. 4                      D. 2                      E.  $-3$

21. Store  $A$  sells 6 pencils for  $\$3.90$ . Store  $B$  sells 8 pencils for  $\$4.48$ . Store  $C$  sells a dozen pencils for  $\$6.96$ . Store  $D$  sells 2 pencils for  $\$1.20$ . Store  $E$  sells 5 pencils for  $\$2.95$ . Which store has the cheapest units rate per pencil?  
 A. Store  $A$                       B. Store  $B$                       C. Store  $C$                       D. Store  $D$                       E. Store  $E$

22. Megan and Beth are asked to paint a mural. Megan can paint the mural by herself in 10 hours. Beth can paint the mural by herself in 15 hours. If they work together, how long will it take the girls to paint the mural?  
 A. 5 hours                      B. 4.5 hours                      C. 7 hours                      D. 7.5 hours                      E. 6 hours

23. Using a coordinate plane, what is the product of the coordinates of the point  $\frac{3}{4}$  of the distance from the origin to the point  $(8, 4)$ ?  
 A. 9                      B. 8                      C. 18                      D. 12                      E. 24

24. In Canada today, the temperature reached  $37^\circ F$ . In Florida today, the temperature reached  $96^\circ F$ . How much hotter was it in Florida than in Canada today, in degrees Celsius?  
 A.  $15^\circ C$                       B.  $27^\circ C$                       C.  $11^\circ C$                       D.  $23^\circ C$                       E.  $20^\circ C$

25. Which formula gives the  $n^{\text{th}}$  term of the sequence?  $-1, 2\frac{1}{2}, 8, 15\frac{1}{2}, \dots$   
 A.  $\frac{n^2+2n-3}{2}$                       B.  $\frac{3n^2+n-1}{2}$                       C.  $\frac{2n^2-2n+1}{2}$                       D.  $\frac{2n^2+n-5}{2}$                       E.  $\frac{n^2+n+3}{2}$

26. If  $0.\overline{783}$  is written as a fraction in lowest terms, how much greater is the denominator than the numerator?  
 A. 214                      B. 107                      C. 223                      D. 109                      E. 91

27. Anita, Bethany and Cali have a total of 293 baseball cards. Anita and Bethany have a total of 176 baseball cards. Bethany and Cali have a total of 208 baseball cards. How many more cards does Cali have than Anita?

- A. 32                      B. 28                      C. 18                      D. 58                      E. 19

28. If one plantain + 2 tacos = \$7.50 and two plantains + 1 taco = \$6.00, how much is 6 plantains + 6 tacos?

- A. \$27.00                      B. \$24.50                      C. \$24.00                      D. \$32.50                      E. \$30.00

29. Two digits are selected from the set {1, 2, 3, 4} to form a 2-digit number. What is the sum of all possible two-digit numbers, if digits may not be repeated?

- A. 330                      B. 317                      C. 346                      D. 370                      E. 358

30. A triangle has side length of 25 inches and 18 inches. What is the largest possible integral length of the third side?

- A. 44 inches                      B. 43 inches                      C. 42 inches                      D. 43.5 inches                      E. 46 inches

31. Set  $A$  has  $n$  subsets and set  $B$  has  $p$  subsets. If  $n - p = 12$ , how many elements are in set  $A$ ?

- A. 24                      B. 16                      C. 8                      D. 12                      E. 4

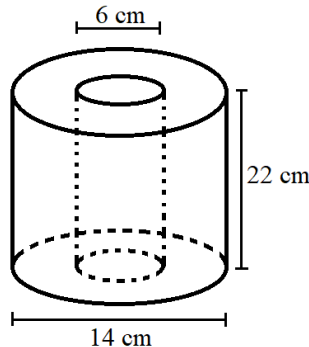
32.  $\overline{AB}$  has endpoints  $A(-9, 3)$  and  $B(7, 7)$ .  $\overline{AB}$  is extended through point  $B$  to point  $C$ . If  $BC = \frac{1}{4}AB$ , what are the coordinates of point  $C$ ?

- A. (9, 6)                      B. (13, 9)                      C. (12, 10)                      D. (11, 8)                      E. (12, 11)

33. If  $f(x) = (x - 4)^2 + 6$  and  $g(x) = x^3 + 7$ , which of the following statements is true?

- A.  $f(5) = g(5)$                       B.  $f(-3) = g(3)$                       C.  $f(9) = g(2)$                       D.  $f(7) = g(2)$                       E.  $f(11) = g(8)$

34. Javier made a plastic tube for a school project. The tube is a cylinder with a diameter of 14 cm with a hollow middle in the form of a cylinder with a diameter of 6 cm. What is the total surface area of Javier's tube (let  $\pi = 3$ )?



- A.  $924 \text{ cm}^2$                       B.  $1,956 \text{ cm}^2$                       C.  $1,440 \text{ cm}^2$                       D.  $1,560 \text{ cm}^2$                       E.  $1,380 \text{ cm}^2$

35. How many positive three-digit integers can be written in such a way that the first digit is greater than the sum of the second and third digits?

- A. 165                      B. 281                      C. 204                      D. 220                      E. 147

36. What is the sum of the first 9 terms of the sequence 2, 3, 5, 10, 18, 33, ...?

- A. 338                      B. 417                      C. 493                      D. 380                      E. 450

37. What is the equation of the circle with center located in the first quadrant and is tangent to the lines  $x = -2$ ,  $y = 6$  and  $x = 8$ ?

- A.  $(x - 4)^2 + (y - 9)^2 = 36$                       B.  $(x + 3)^2 + (y + 7)^2 = 9$                       C.  $(x - 3)^2 + (y - 6)^2 = 16$   
 D.  $(x - 3)^2 + (y - 11)^2 = 25$                       E.  $(x - 4)^2 + (y - 6)^2 = 36$

38. What is the area of a quadrilateral with vertices located at  $(6, 4)$ ,  $(-6, -3)$ ,  $(-9, 2)$  and  $(8, -5)$ ?

- A.  $58 \text{ units}^2$                       B.  $29 \text{ units}^2$                       C.  $106 \text{ units}^2$                       D.  $80.5 \text{ units}^2$                       E.  $101.5 \text{ units}^2$

39. The 4<sup>th</sup> term of a sequence is  $-5$ , the 5th term of the sequence is  $-1$ , and the 7<sup>th</sup> term of the sequence is  $-6$ . Every term after the 3<sup>rd</sup> term is the sum of the three preceding terms. What is the value of the 14<sup>th</sup> term of this sequence?

- A.  $-288$                       B.  $-373$                       C.  $-327$                       D.  $-314$                       E.  $-292$

40.  $5! \times 9! = 10! \times$  \_\_\_\_\_

- A.  $8!$                               B.  $16$                               C.  $12$                               D.  $8$                               E.  $4!$

41. The product of two consecutive positive integers is  $702$ . What value is seventeen more than the smaller integer value?

- A.  $44$                               B.  $53$                               C.  $54$                               D.  $43$                               E.  $32$

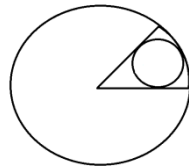
42. If  $\left(\left(\frac{2a^3b^4}{8a^{-4}b^{-1}}\right)\left(\frac{32a^{-7}b^{-2}}{8a^3b^3}\right)\left(\frac{a^{-4}b^{-2}b^6}{a^{-5}}\right)\right)^2 = \frac{b^m}{a^n}$ , then what is the value of  $7n - 3m$ ?

- A.  $6$                               B.  $4$                               C.  $-10$                               D.  $-12$                               E.  $8$

43. What is the product of the roots of the cubic equation  $2x^3 - 7x^2 - x + 12 = 0$ ?

- A.  $7$                               B.  $\frac{-7}{2}$                               C.  $\frac{2}{7}$                               D.  $-\frac{1}{6}$                               E.  $-6$

44. A small circle is inscribed in a  $60^\circ$  sector of a larger circle, so the small circle is tangent to all 3 sides of the sector. What is the ratio of the area of the small circle to the area of the large circle?



- A.  $\frac{1}{12}$                               B.  $\frac{1}{18}$                               C.  $\frac{1}{6}$                               D.  $\frac{1}{9}$                               E.  $\frac{1}{15}$

45.  $4 \log_2 8 - 4 \log_2 4 =$  \_\_\_\_\_

- A.  $2$                               B.  $4$                               C.  $8$                               D.  $16$                               E.  $64$

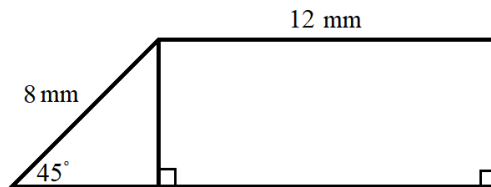
46. Lucinda has a sculpture that is currently worth  $\$3,000$ . Because the paint is fading away, the sculpture's worth is depreciating at a rate of  $40\%$  per year. What will be the value of the sculpture after 2 years from now?

- A.  $\$480$                               B.  $\$960$                               C.  $\$1,080$                               D.  $\$1,240$                               E.  $\$1,560$

47. The area of a circle is  $64\pi \text{ cm}^2$ . In terms of  $\pi$ , what is the length of a  $45^\circ$  arc of the circle?

- A.  $2\pi \text{ cm}$                               B.  $4\pi \text{ cm}$                               C.  $16\pi \text{ cm}$                               D.  $12\pi \text{ cm}$                               E.  $8\pi \text{ cm}$

48. What is the area of the trapezoid?



- A.  $16 + 24\sqrt{2} \text{ mm}^2$                       B.  $16 + 48\sqrt{2} \text{ mm}^2$                       C.  $16 + 192\sqrt{2} \text{ mm}^2$                       D.  $16 + 96\sqrt{2} \text{ mm}^2$                       E.  $16 + 144\sqrt{2} \text{ mm}^2$

49. What is the product of the solutions of the equation  $\sqrt{6n - 38} + 2 = n - 3$ ?

- A.  $72$                               B.  $56$                               C.  $63$                               D.  $54$                               E.  $-48$

50. The value of  $14x + 98y$  is equal to  $126$ . What is the value of  $17x + 119y$ ?

- A.  $145$                               B.  $153$                               C.  $148$                               D.  $165$                               E.  $142$

2018 – 2019 TMSCA Middle School State Mathematics Championship Test Answer Key

1. D	18. C	35. A
2. A	19. E	36. E
3. E	20. B	37. D
4. B	21. B	38. E
5. E	22. E	39. A
6. A	23. C	40. C
7. C	24. A	41. D
8. B	25. D	42. B
9. B	26. B	43. E
10. B	27. A	44. D
11. A	28. A	45. B
12. A	29. A	46. C
13. D	30. C	47. A
14. C	31. E	48. B
15. A	32. D	49. C
16. B	33. D	50. B
17. E	34. D	

2018-2019 TMSCA Middle School State Mathematics Championship Test Selected Answers

19. Converting all gives us  $1.2 \text{ km} = 1,200 \text{ meters}$ ,  $10,000 \text{ cm} = 100 \text{ meters}$  and  $120,000 \text{ mm} = 120 \text{ meters}$ . Therefore,  $1.2 \text{ km} - 10,000 \text{ cm} - 120,000 \text{ mm} = 1,200 - 100 - 120 = 980 \text{ meters}$ .

23. The point  $\frac{3}{4}$  of the distance from the origin to the point  $(8, 4)$  is  $(\frac{3}{4}(8), \frac{3}{4}(4)) = (6, 3)$ . The product of the coordinates is  $6(3) = 18$ .

28. Let  $p = \text{plantain}$  and  $t = \text{taco}$ . We have the equations  $p + 2t = 7.50$  and  $2p + t = 6$ . If we add the two equations together, we get  $3p + 3t = 13.50$ . Doubling all gives us  $6p + 6t = \$27.00$ .

29. There are 12 two-digit numbers that can be created. They are 12, 13, 14, 21, 23, 24, 31, 32, 34, 41, 42, and 43. Their sum is then  $12 + 13 + 14 + 21 + 23 + 24 + 31 + 32 + 34 + 41 + 42 + 43 = 330$ .

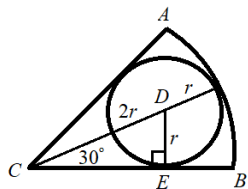
31. The number of subsets of a set with  $x$  elements is found by  $2^x$ . So, we are looking for sets to have subsets that differ by 12, thus  $2^m - 2^p = 12$ . The only powers of 2 that differ by 12 are  $2^4$  and  $2^2$ , because  $2^4 - 2^2 = 16 - 4 = 12$ . Since  $2^4$  represents all the subsets of set  $A$ , set  $A$  has 4 elements.

35. We are asked to find the number of positive three-digit integers can be written in such a way that the first digit is greater than the sum of the second and third digits. If we start with 1 as our first digit, there is only one number that satisfies our condition, which is 100. If we start with 2 as our first digit, then there are three numbers that satisfy our condition, which are 200, 201 and 210. If we start with 3 as our first digit, then there are 6 numbers, which are 300, 301, 302, 310, 311 and 320. If we follow this pattern, there are  $1 + 3 + 6 + 10 + 15 + 21 + 28 + 36 + 45 = 165$  numbers that have their 1<sup>st</sup> digit greater than the sum of the 2<sup>nd</sup> and 3<sup>rd</sup> digits.

40. Rewrite  $5! \times 9! = 10! \times n$  as  $n = \frac{5! \times 9!}{10!}$ . Therefore,  $n = \frac{(5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)(9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)}{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{10} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 2} = 12$ .

42.  $\left( \left( \frac{2a^3b^4}{8a^{-4}b^{-1}} \right) \left( \frac{32a^{-7}b^{-2}}{8a^3b^3} \right) \left( \frac{a^{-4}b^{-2}b^6}{a^{-5}} \right) \right)^2 = \left( \left( \frac{a^7b^5}{4} \right) \left( \frac{4}{a^{10}b^5} \right) \left( \frac{ab^4}{1} \right) \right)^2 = \left( \frac{4a^8b^9}{4a^{10}b^5} \right)^2 = \left( \frac{b^4}{a^2} \right)^2 = \frac{b^8}{a^4}$ . Thus,  $m = 8$  and  $n = 4$ . Therefore,  $7n - 3m = 7(4) - 3(8) = 28 - 24 = 4$ .

44. Look at the sector and label as below,



Draw an angle bisector of  $\angle ACB$ , which will also include the diameter of the small inscribed circle. Let the radius of the small circle be  $r$ . Since  $m\angle ACB = 60^\circ$ ,  $m\angle DCE = 30^\circ$  and  $\triangle DEC$  is a 30-60-90 right triangle. If  $DE = r$ , then  $CD = 2r$  and the diameter of the large circle is  $3r$ . The area of the small circle is  $\pi r^2$  and the area of the large circle is  $9\pi r^2$ . Therefore, the ratio of the area of the small circle to the

of the large circle is  $\frac{\pi r^2}{9\pi r^2} = \frac{1}{9}$

45. Since  $n \log_a b = \log_a b^n$ ,  $4 \log_2 8 = \log_2 8^4$  and because  $8 = 2^3$ ,  $\log_2 8^4 = \log_2 (2^3)^4 = \log_2 2^{12} = 12$ . Similarly,  $4 \log_2 4 = \log_2 4^4 = \log_2 (2^2)^4 = \log_2 2^8 = 8$ . Therefore,  $4 \log_2 8 - 4 \log_2 4 = 12 - 8 = 4$ .

50. We are given that  $14x + 98y = 126$ . We can divide entire equation by 14 and  $\frac{14x}{14} + \frac{98y}{14} = \frac{126}{14} \rightarrow x + 7y = 9$ . Now, multiply the entire equation  $x + 7y = 9$  by 17 and  $17(x + 7y) = 153 \rightarrow 17x + 119y = 153$ .